A Critique of Paper-and-Pencil Secondary One Mathematics Semestral Assessment Papers

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Abstract
Assessment is an essential part of the teaching and learning process. Since assessment plays such a significant part in the educational process, it is imperative that we examine closely the individual questions that make up the assessment paper. This paper is based on a preliminary investigation into about 30 sets of secondary one semestral examination papers. In this paper only a sample of problematic short-ended and structured / long-answer questions have been highlighted and discussed. From this sample, it could be seen that short-ended and structured / long-answer questions are challenging and demanding to design. The questions were scrutinized based on the following criteria involved in the designing of test/examination questions: (1) Questions with incorrect Mathematical Concept, (2) Questions which test trivial facts, (3) Questions with ambiguous verbal communication, (4) Questions with inconsistent symbols and notations, (5) Questions with misleading diagrams, (5) Questions with impracticality of scenarios, and (6) Questions with imbalanced learning objectives.

Keywords: Assessment, Secondary One, and Criteria

Introduction

Assessment is an essential part of the teaching and learning process (MOE, 2004). It provides information to the teacher on the extent to which learning objectives have been achieved by students and the extent to which teacher has been effective as a facilitator of learning. Based on the results of the assessment, the teacher is able to decide whether to proceed to the next teaching topics or modify teaching method or provide remedial lessons or give enrichment exercises. The results of the assessment also provide feedback to students on the effectiveness of their learning.

While a list of alternative assessment has been advocated, the paper-and-pencil assessment is still the most common assessment technique utilise in the classroom. Thus the ability to assess mathematics questions according to some rigorous criteria is an important skill that teachers and examination setters need to acquire. This will ensure that examination questions are continuously fine-tune so that they are reliable, valid and fair to the target groups of students. Questions that are poorly designed have two major drawbacks. Firstly, they may not measure reliably and validly the intended learning outcomes. Students may be bewildered by the defects in the questions that they are unable to show their level of competence. Students may get frustrated easily when they think the questions are "unfair" to them, being too difficult, ambiguous, confusing, or have not been covered in their mathematics lessons. Secondly, questions that have faulty mathematics will perpetuate these errors among students as well as teachers who are not aware or mathematically less competent. The effect is even more

pronounced when questions with incorrect mathematics are originated in the school examinations. Some mathematics teachers tend to modify past examination questions by changing the numbers or context, believing that what have tested in national examinations, textbooks, workbook and assessment books are perfectly correct, beyond questioning. This uncritical frame of mentality ought to be disputed. Since assessment plays such a significant part in the educational process, it is imperative that we examine closely the individual questions that make up the assessment paper. This paper is based on a preliminary investigation into about 30 sets of secondary one express stream semestral assessment papers of different secondary schools. Each set of semestral assessment paper typically comprises short-ended and structured / long-answer questions. Some short-ended and structured / long-answer questions are highlighted and discussed in this paper. A short-ended question has a stem. The stem poses the task or problem and can take the form of a direct question or an incomplete statement. It should note that while there were examination questions which were problematic, there were also many well-designed examination questions. These examination questions had effectively test students' mathematical content, skills and acquisition of mathematical concepts which were also fully aligned with Singapore mathematical curriculum goals. The questions were scrutinized based on the following criteria involved in the designing of examination questions:

1. Questions with incorrect Mathematical Concept;
2. Questions which Test Trivial Facts;
3. Questions with ambiguous verbal communication;
4. Questions with inconsistent symbols and notations;
5. Questions with misleading diagrams;
6. Questions with impracticality of scenarios; and
7. Questions with imbalanced learning objectives (Yeo, 2004).

The criteria suggested above are not exhaustive but should be comprehensive enough to cover the main ideas to bear in mind when designing test questions. The potential problems arising in the seven identified criteria are discussed below, together with some approaches that would help to improve them.

1. Questions with Incorrect Mathematical Concept

The main consideration in this category is: Are the mathematics concepts and rules used in the question correct? A common phenomena is related to the understanding of π. Examine the following question:

Example 1

A wire of length 1.8 m is cut into two pieces, one forming a circle of radius 14 cm and the other forming a square. Find the length of the square. ($\pi = \frac{22}{7}$)

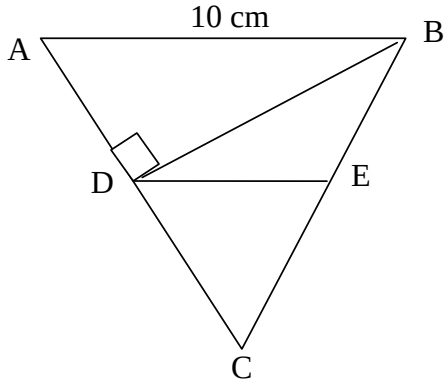
The above question indicates that π is a rational number, which is incorrect. The correct way to phase the above question is to write "Take π to be $\frac{22}{7}$", or "Take π to be 3.14". Twenty-six prospective secondary mathematics teachers were told to write down their understanding of π. The responses from the prospective secondary mathematics teachers were varied. About 40% of

the prospective secondary mathematics teachers stated that $\pi = \frac{22}{7}$ or $\pi = 3.14$. A third of them mentioned some vague statements such as "it is used in finding the area of circle, semi-circles and quadrants" and "the formula for the area of the circle required the value of π". Only seven of them wrote π is equal to circumference divided by diameter. Although all the prospective secondary teachers could apply the value of π in computing the area of circle, some of them had the misconception about the value of π. One of the interesting relationships that students can discover in mensuration is that between the circumference of a circle and the length of the diameter. Students could measure the diameter and circumference of several circular objects and use calculators to compare these two measures. The circumference of every circular object is approximately 3.14 times as long as the diameter. The exact ratio is an irrational number close to 3.14 and is represented by the Greek letter π.

Example 2

ABC is an equilateral triangle with AB = 10 cm. BD is perpendicular to AC and the area of triangle ABC is 45cm^2.

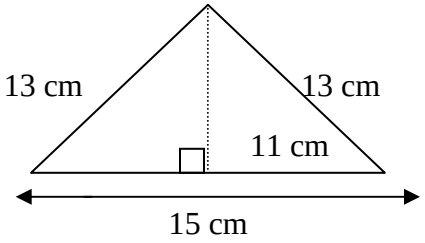
Find (a) the length of BD ,
(b) the area of triangle BED if D and E are the midpoints of sides AC and BC respectively



Example 3

The cylindrical wax candle has a volume of 1650 cm^3. Given that it has a height of 21 cm, calculate the radius of the wax candle.

Given that the candle is melted and poured into a triangular prism mould with cross section as shown, calculate the height of the wax in the triangular prism. Hence, calculate the surface area of the prism in contact with wax. (Take $\pi = 3.142$)



Although the stems and the questions in example 2 and 3 are correct, the two diagrams are incorrect. The two right-angled triangles are drawn such that the dimensions do not satisfied Pythagoras Theorem. Teachers who set questions involving right-angled triangles should always verify using the Pythagoras Theorem.

2. Questions which Test Trivial Facts

Questions that emphasises on trivial facts rather than mathematics concepts or processes could hinder the fostering of higher order thinking in students. This type of question simply requires the students to recall the facts that has been discussed or worked out in class. However, this is not to mention that testing on facts is not to be encouraged at all.

Example 4

Write down the next term for the following number sequences.

(a) 2, 5, 8, 11, ___

(b) 1, 1, 2, 3, 5, 8, ___.

Example 5

Complete the following equivalent ratios

(a) 3 : 5 = 9 : ☐ (b) ☐ : 7 = 12 : 21

Example 6

Find the value of

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$$

Example 4 shows that the student would simply add 3 to get the next term for part (a) and add 4 to find the answer in part (b) while example 5 is a knowledge item which the students are able to find the ratios easily. As for example 6, it is one routine step solution which the students have been solving such problem in their primary years. In fact, the three examples are considered as trivial because it is regarded as common knowledge, is merely a recall exercise. Although example 4, 5 and 6 do not assess major learning outcomes in mathematics, this type of questions is best tackled through meticulous vetting and stringent quality control throughout the examination setting process. Setters and vetters of examination questions should be provided with clear guidelines on the expected question type profile for each examination paper.

3. Questions with ambiguous verbal communication

The main issues raised by most students are: "*What are the implicit assumptions made?*" and "*Are the students expected to know all these assumptions?*" Questions that view by another students or teachers to be ambiguous may not be so to the students who have the experience of making these assumptions in their classwork. Although it is difficult to be extremely precise in the use of daily and mathematical language, the context in the question will generally provide a indication of the implicit assumptions made. For instance, example 7 assumes that the average means arithmetic mean and one week refers to five-day week. In example 8, part (b) and part (c) should be rephrased as a question instead of a direct statement while example 9 should be more precise as round off to the nearest units is not familiar to most students.

Example 7

During one week, the time , in minutes , which Mr. Tan took to travel to work were 25 ,30 , 21, 23, 31 . Calculate his average time, in minutes.

Example 8
A bicycle wheel has a diameter of 70 cm.

(a) Find the circumference of the wheel. (Taking $\pi = \frac{22}{7}$)

(b) The distance traveled by the bicycle after the wheel has made 250 revolutions. (Give your answer in km)

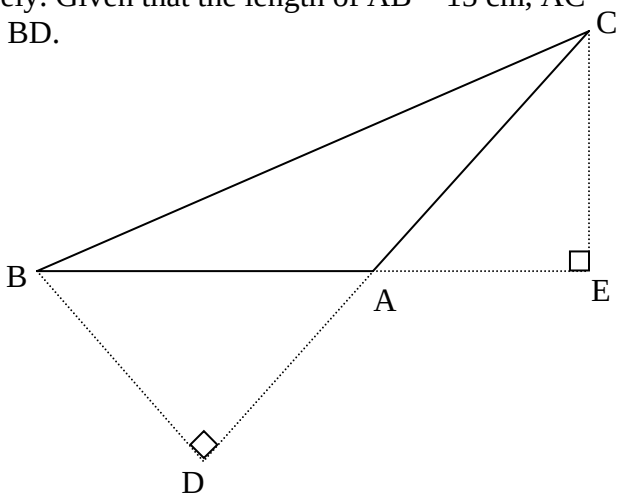
(c) The average speed of the bicycle if it takes 2 min to complete the journey. (Give your answer in km/hr)

Example 9
Round off 2599.841 to the nearest units.

Example 10
A farmer has 24 cows. He has enough fodder to feed them for 30 days. If he has bought 12 more cows before feeding day, how many days can the fodder last?

The context in example 10 may not be meaningful to most of our students in Singapore as they have no experience in farming. Moreover, word like fodder is uncommon and it should also be avoided in a semestral assessment papers. A mathematics semestral assessment should not be an assessment of English. This can be accomplished by using simple sentence structures and vocabulary as well as context that is meaningful to students.

Example 11
In the figure shown below, BD and CE are perpendicular to CA and BA produced respectively. Given that the length of AB = 15 cm, AC = 12 cm and CE = 8 cm, find the length of BD.



In example 11, it is not indicated in the question that the two triangles BCE and BCD are congruent. Students may find it difficult to solve this problem and they may spend too much time on this question during examination. This may create unnecessary test anxiety during examination. To avoid such situation, the essential conditions and the data in any test item should be provided so that students will not make any wrong assumptions or wild guesses.

4. Questions with Inconsistent Symbols and Notations

Mathematical notations and symbols are generally precise. In the secondary mathematics syllabus, certain symbols have specific meanings, such as $\angle$ is used to denote the angle and the symbol $\equiv$ means 'is congruent to'. Different textbooks have different ways of telling and writing time using 24-hour clock. In Singapore secondary schools, all students are taught to see that in the notation for 24-hour clock, there is a space after the first two digits and no unit is required. For example: 08 25, a space is left between the digit 8 and 2. A less obvious problem relates to this criteria is the use of box as a blank. In example 12, it is redundant to have a question mark in the box.

Example 12

Put in the correct sign, '<', '>' or '=' into the box.

$$\frac{1}{4} \quad \boxed{?} \quad \frac{1}{3}$$

Any symbol or notation printed on the semestral assessment paper must be clear and legible. For instance, the division sign, $\div$, is easily confused with the addition sign. Numbers and symbols in superscript should be big enough.

5. Questions with misleading diagrams

Many questions with diagrams are only sketches and not accurately drawn. This is one of the implicit assumptions that students need to be aware in the examination. However, there will be situations where the students incline to treat a diagram as exact. It is necessary to inset a qualifying phrase to indicate the diagram is "not drawn to scale" or "not accurately drawn".

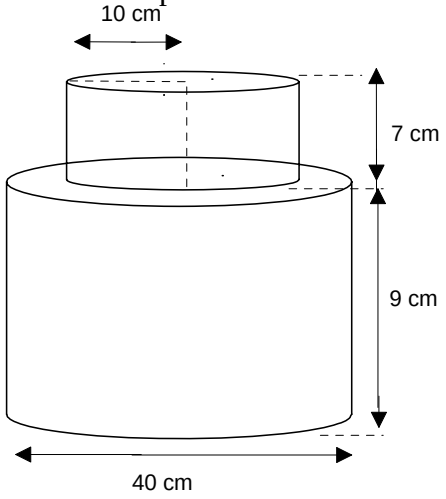
Example 13

Penny bakes a 2-layered cake comprising 2 cylindrical shapes as shown in the diagram. If she eats $\frac{1}{4}$ of the top layer, find

(a) the volume of the remaining cake,

(b) the surface area of the remaining cake.

[Take π to be 3.142.]



Even a sketch should be reasonably accurate or the dimensions must be in the correct proportions. In example 13, the diagram shows that the height and the radius are not in the correct proportion. Some diagrams may be mathematically impossible or scientifically inaccurate. Teachers should be cautious and avoid making such flaws. Some questions require students to read and measure a length or angle on the given diagram. It is essential to specify the degree of accuracy in the measurements.

6. Questions with Impracticality of Scenarios

In setting any semestral assessment paper, teachers will modify some questions from the past-year national examination papers. They may just change the numerical values in certain standard questions. It may not be desirable in certain situations as the data used are not realistic. In addition, the answer computed by the students is not reasonable in real life context. Below showed five examples that were set in the semestral assessment paper.

Example 14

A bicycle wheel has a radius of 25 cm. The wheel turns through 3500 rounds when Gong Gong rides on it. How far did he cycle? Give your answer in metres. (Take π to be $\frac{22}{7}$)

Example 15

During the Great Singapore Sale, David bought 21 hi-fi systems and 18 amplifiers for $25 410. Find the cost of each amplifier if each hi-fi system costs $766.

Example 16

A flight departed from Perth at 1435 and was expected to reach Singapore at 2115 on the following day. How long was the flight?

Example 17

Mr. Lim exchanged S$11600 for US dollars at a rate of S$1.45 to US$1 for his trip last year. He spent US$6800 and exchanged the remaining US dollars back into Singapore dollars at a rate of S$1.43 to US$1. Calculate the amount of Singapore dollars he got back, giving your answer correct to the nearest ten dollars.

From example 14 to 17, the data in the questions is unrealistic. For example 14, it is very challenging for Gong Gong (grandfather) to cycle a bicycle when the wheel turns through 3500 rounds. Gong Gong may not have the stamina to achieve it. While in example 15, it is unlikely for an individual to purchase so many hi-fi systems and amplifiers unless David is a retailer who sells hi-fi systems and amplifiers. Although some secondary one student may not aware that the duration from Perth to Singapore may take less than 6 hours, teachers need to ensure that the duration to be computed in example 16 is reasonable. Finally, the exchange rate for Singapore and US dollars seldom occurs at a rate of S$1.45 to US$1 and S$1.43 to US$1 as shown in example 17..

Example 18

It is given that 12 men will take 30 days to complete building a bungalow. Find the number of days needed for 15 men to complete building 3 bungalows.

In our Singapore context, it is rather uncommon for an ordinary Singaporean to see a bungalow in our housing estate. Moreover, in real life situations, it may need more than 15 men to build 3 bungalows that are stated in the question. It is critical to use context that is known, relevant and meaningful to the target students. Teachers should adapt the question by modifying the context or story line. In addition, the units that are stated in the questions should be familiar to the students. Otherwise, it will add another aspect of difficulty that is not the intent of the question is set to assess.

7. Questions with Imbalanced Learning Objectives

The test questions must cover a range of lower to high-order skills so that students of different abilities and interests can show what they are capable of doing. Although it is necessary to set some high order thinking questions in the semestral assessment, the questions in the semestral assessment paper must assess only objectives stated in the syllabus or taught in lessons. Below showed an example that was found in the semestral assessment paper:

Example 19

Given that $a + b + c = 12$

$a + b + d = 11$

$a + c + d = 10$

$b + c + d = 9$

find the value of a, b, c and d.

Solving simultaneous equations involving four unknowns are challenging for most secondary one student. Secondary one student only solves linear equation with one unknown. Most of the students may not have the techniques and procedures of solving simultaneous equations with four unknowns as it is not in the secondary one syllabus. Even majority of secondary three and four students will have difficulty finding the four unknowns and let alone be secondary one students.

It will be useful to construct a Table of Specifications which gives an overall view of the content to be assessed. It also helps to ensure a broad and balanced coverage of the syllabus taught. One of the concerns is that teachers have the tendency to set and modify questions such that students will have difficulty in solving the problem correctly. They tend to find out what the students do not know rather than what they do know. This will discourage the students from learning something worthwhile from the lessons and tests.

Conclusions

In this paper only a sample of problematic short-ended and structured long questions have been highlighted and discussed. From this sample, it could be seen that short-ended and structured / long-answer questions are challenging and demanding to design. It could also be seen that short-ended questions may not be useful in testing certain more complex concepts. In fact, the inappropriate use of short-ended and structured / long-answer questions could hinder student learning or could result in student misconceptions or weak conceptions through missing out the whole concept. In Singapore, students spent a lot of time preparing for their tests and examinations. If semestral assessment questions are not well designed, such as those examples shared in this paper, then it will not be effective for the intended goals. Instead, it may mislead the students. It is also unfair for the students who are proficient in the content but are misled because of poorly designed questions. We must acknowledge that it is a challenging task to design questions that are new and original as well as assessing concepts and skills. Nevertheless, schools may come together to network and pool resources, share ideas for this challenging and demanding task.

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