How logical are the logic questions in IQ tests?

Yagub N. Aliyev

School of IT and Engineering, ADA University
yaliyev@ada.edu.az

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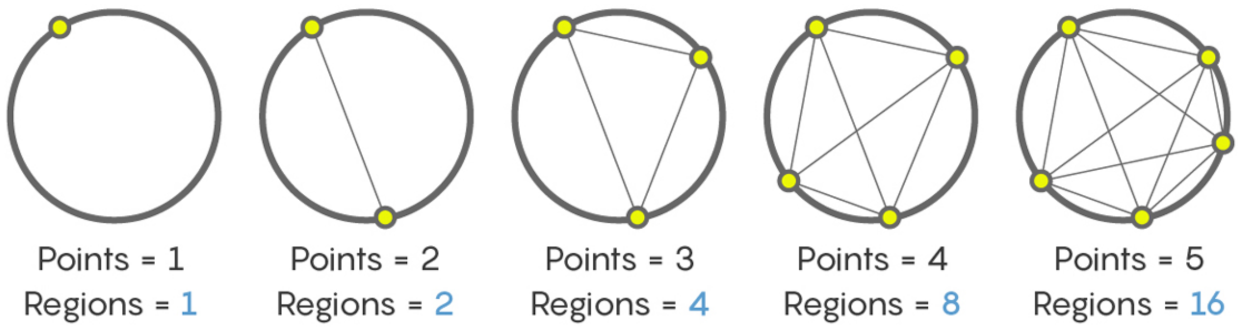
Abstract. In the paper, we discuss some of the logic questions used in various exams, tests and quizzes. These logic questions were originally used in IQ tests measuring psychological condition or intellectual level of people under study. Nowadays these tests are very popular in all levels of education: from primary school to PhD level. The paper will give a constructive criticism of the problem types used for such IQ tests. The paper contains many recommendations for the improvement of these tests and their quality. We separate two types of questions: 1) those which are deterministic – problems that you can solve using mathematical knowledge and skills, and 2) probabilistic – problems which can be solved with the presence of some luck or chance. We recommend not to use such incomplete problems of the second type in important exams and tests. The logic questions containing elements of chance can be used for recreational purposes but entrance or graduation exams, tests for job applicants is not a correct place to use such questions. We will also discuss negative effects of the use of the questions of the second type on school children and their understanding of mathematics.

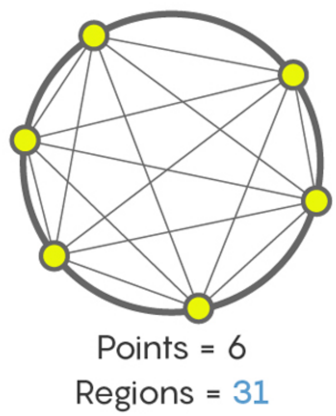
Introduction. Logic questions that appear in now very popular IQ tests lie usually at the periphery of mathematician’s interests. As any normal mathematician, I would rather ignore them as something exotic or secondary than look at them as a research topic. But this year some very important changes happened, and it made me look at them from a different perspective. Four of my children participated in some kind of exams and olympiads containing such questions. Sometimes, I was unable to solve these problems meant for school children and it made me feel bad not just for myself and my children but also for all those poor kids around the world who were forced to go through these unnecessary obstacles which has no basis in established educational system. I went through various types of questions and found their main shortcomings. All these questions contain an element of luck and they are in some sense incomplete. In a certain sense, you can go from one side to the other very easily and this gives an illusionary credibility for the question but to find a way back is sometimes very problematic and one can do this only if the solver is sufficiently lucky. Usually, problem proposers ask exactly these inverted questions. On the other hand, the teachers who prepare these questions are not always professional educators with solid mathematical background. In some cases, these problem composers are not even aware that they are doing something wrong and harmful. I also want to mention the fact that the teachers who prepare and consult the students who are going to participate in such exams sometimes lack deep mathematical knowledge and sometimes their choice of material is misleading for the students and their parents. What I will present here is my professional mathematician’s opinion and I am very well aware that people from other fields of education can have different and even opposing opinion about the subject.

1. *Problems about Sequences:*

I. What is wrong with these questions? I would like to start with an example. Take this sequence of numbers: 1, 2, 4, 8, 16, … . What do you think, which number should come next? Majority of the people, familiar with school mathematics (the topic is geometric series), will say “Of course, it is 32! Each next number is the double of the previous”. But if you look into the problem deeply you might question your choice and ask yourself “Why only 32?”. Can’t we find a different scenario which takes the sequence in a different direction? Yes, we can and you don’t need to invent something extraordinary to find that logic. Look at the following two pictures which is taken from [1] with permission.



As we can see the segments joining the given points divide the circular region into several regions. How many regions are there if the number of points is 6?



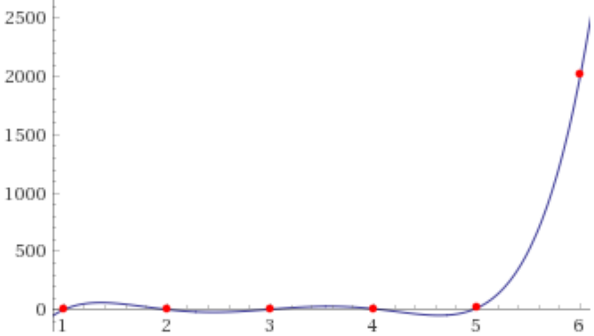
Didn’t expect to see here 31, did you? Yes, me too. I actually counted one by one to be sure. So, 31 is also a very logical candidate for the next term in the sequence 1, 2, 4, 8, 16, … . We can give many examples of this type and each of them is sufficient to discredit the problems of this type in IQ tests especially those appeared in exams which have life changing impacts for the students. In fact it is possible to show that any number, not just 31 and 32, not even integer or positive, is possible instead of the question mark in the sequence 1, 2, 4, 8, 16, ?. There is a theorem in mathematics proved by Lagrange which says that one can find a formula for any finite sequence of numbers. Without going into details, I will give here a simplified version of this theorem. Suppose you have numbers a, b, c, d, e, f and you want to find a formula (Lagrange’s polynomial) such that $p(1) = a,\ p(2) = b,\ p(3) = c,\ p(4) = d,\ p(5) = e,\ p(6) = f$. If you accept a formula here as “logic” then Lagrange’s Theorem can help us to construct a logic behind any sequence of 6 numbers a, b, c, d, e, f. In particular, we can find a logic to continue the given sequence with number 2019 like this 1, 2, 4, 8, 16, 2019. The last number, the year in which this conference takes place is probably the last number that you would expect to see in this sequence but here is a formula for it:

$$-1987+\frac{272311\,x}{60}-\frac{1}{24}(89437\,x^2)+\frac{1}{12}(16895\,x^3)-\frac{1}{24}(5963\,x^4)+\frac{1}{30}(497\,x^5)$$

or a version more suited for calculations

$$-1987+x\left(\frac{272311}{60}+x\left(-\frac{89437}{24}+x\left(\frac{16895}{12}+x\left(-\frac{5963}{24}+\frac{497\,x}{30}\right)\right)\right)\right)$$

This polynomial was calculated by Wolfram Alpha [2]. Here is a graph (also generated by Wolfram Alpha) of the polynomial that indicates the set of points through which the curve passes: {(1,1), (2,2), (3,4), (4,8), (5,16), (6, 2019)}.



I don't expect that this argument will stop the composers who fill their books with problems about sequences produced in industrial size. At least after these arguments the composers of these problems will know that not everyone approves what they do. There is a chapter of 30 pages in the book [3] which is densely populated by the problems of this type. It is a logic problem book very popular among students preparing for exams formerly known in Azerbaijan and Turkey as YÖS. You can only imagine what a mathematician working on Interpolation Polynomials thinks about each of those question marks in the chapter.

II. How can we improve them? Our criticism of the problems about sequences can form an incorrect opinion that this topic is completely inappropriate for a logical test. Quite the opposite is true. There are many types of questions about sequences which are more than logical and mathematically sound. For example, problem composers can ask finding a correct formula expressing a sequence.

Problem. Which of the following formulae best describes the sequence of numbers 1, 3, 6, 10, 15,…?

A) n^2
B) $2n-1$
C) $2^n-(n-1)!$
D) $\frac{n(n+1)}{2}+(n-1)(n-2)(n-3)(n-4)$
E) $\frac{n(n+1)}{2}$

The variant E is the correct answer. The others describe the sequence only up to a certain point. If the composers will use the properties of the sequences that can be mathematically substantiated and do not leave anything ambiguous in the statement of the questions then the topic of sequences is a very fruitful field to gather interesting logical gems. We will stop here the discussion of sequences to look at another type of incomplete problems.

2. ***Problems about Operations***: You have probably seen problems like this before:
Problem. 4∇3=37, 2∇1=7, 3∇2=19, 5∇3=?
The composers use various symbols instead of ∇. Some of them spend much time to find a new symbol instead of ∇ in the list of advanced symbols of their text editor. Instead of this they could concentrate more on the content of the problem and try to make it more solid and flawless. Unfortunately, this type is one of the most illogical question types in the literature. I will try to explain the absurdity using the above example, a variant of which appeared recently in a test for 6th grade students. Some of you have already noticed that $4^3-3^3=37, 2^3-1^3=7, 3^3-2^3=19$. So, it would be "logical" to expect that the answer to the question is $5^3-3^3=98$ but this number wasn't included within the multiple-choice question. Desperate student tried to find a number similar to 98 and the best option seemed to be 49 which is half of 98. Fortunately, this time the student picked a correct answer. In a second you will also "understand" why this "answer" is "correct" but before that I would like to ask you this natural question: *Is this the best way to check mental abilities of the schoolchildren? Why not using a normal math problem instead of this absurdity which is a product of immature mathematical tastes?* You probably can't wait to "understand" why 48 is "the correct" answer. Because, for some reason, unknown to us, the composer thought that

$$4^2+4\cdot 3+3^2=37, \qquad 2^2+2\cdot 1+1^2=7, \qquad 3^2+3\cdot 2+2^2=19$$

was a better explanation for the pattern and therefore "the correct answer" should be $5^2+5\cdot 3+3^2=49$. Again, within this topic it is possible to compose many good problems without any gaps in its statement. Take this popular type for example.
Problem. x∇y=x-y+1, (4∇3)∇(7∇5)=?
In the problem, the two-variable function x∇y is uniquely defined as x-y+1, so the solver doesn't have any difficulty to start solving it. The only important thing is to pay attention to the brackets which should be completed before the operation ∇ in the middle. The solution will look more or less like this: 4∇3=4-3+1=2, 7∇5=7-5+1=3, (4∇3)∇(7∇5)=(2)∇(3)=2∇3=2-3+1=0.
Yes, operations can be a very interesting theme in logic questions but the composer should be careful not to leave anything unclear or poorly defined. The mentioned problem book [3] contains many good problems of this type in pages 85-114. But unfortunately, it contains also hundreds of bad problems of this type in the pages 65-84.

2. ***Problems about Tables***: We can extend our criticism of the previous types to the problems about tables. Let us start again with an example.
Problem.

2	6	3
5	4	2
7	2	?

The flaw of this type of problems is similar to the flaw of the bad problems about sequences. You can put any umber instead of the question mark and then you can find a formula to claim that it is logical to put that number there. If you couldn't solve the problem then you shouldn't blame yourself because it is indeed very illogical question. The official solution will be something along the following lines.
First row: 2*6=12, 1+2=3
Second row: 5*4=20, 2+0=2
Third row: 7*2=14, 1+4=5. Answer: ?=5

We can ask ourselves and the composers the following questions: *What does the problem of this type measure? Is it a merit to find exactly the same logic that the composer had in his or her mind? What if the student found a different pattern but it was not in the list of answers. Whom should we blame in this case? The student or the composer? Why should a student lose points in an exam if he or she can't find the route that the composer chose among infinitely many other possible routes?*
Many math teachers complain that after long exposure to such logic questions the attitude of the students towards mathematics changes and in majority of the cases it is not a positive change. Some students start to understand mathematics as a game where you can do anything that comes to your mind. This is not what we are trying to promote in our math education system. We are trying to promote elite mathematics that can help students solve important problems of science, technology and engineering.
Let us return to the questions about tables. Again, we should say that there are many problems about tables in logic tests which are mathematically correct and complete. For example:
Problem. Find $a + b + c$ if

+	*c*	*d*
a	4	5
b	5	6

The problem can be interpreted as a system of equations

$$a + c = 4,$$
$$a + d = 5,$$
$$b + c = 5,$$
$$b + d = 6.$$

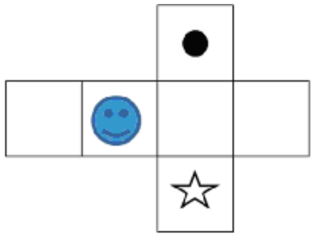
By solving the system, we find that $a = 1$, $b = 2$, $c = 3$, and $d = 4$. Other more elementary solutions are also possible here.

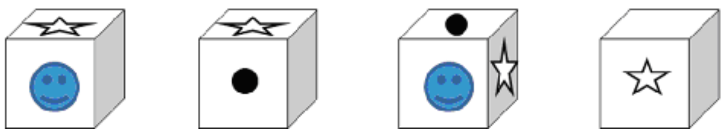
I would like to stop here my criticism and say some positive things about the IQ tests. Some of you are ready, possibly, to bury these logic questions forever and replace them with math problems. But I have to say that if the examiners will be careful with the choice of the material then these logic problems can be very successful for the measurement of intellectual capabilities. There are many types of logic problems that can be used without any hesitation in important exams and tests. These types include *rebus problems* where a certain arithmetical operation is done on multi-digit numbers and then the digits are replaced by letters with the condition that the same letters are denoted by the same numbers and the different numbers by the different letters. One more condition is that the first digit of the multi-digit number can't be a zero. Here is an example similar to the one from [4, page 11]:
Problem. Find $(d + k): k + t$ if

$$\begin{array}{r} 3d32 \\ +\, k1t0 \\ \hline 578k \end{array}$$

Another type is the type containing *3D problems* involving sometimes a die or other solids which the student needs to mentally rotate and move to find the correct answer. For example, the following is taken from [5] with permission.

Problem.





It is necessary to point out in the statement what the above picture describes. In this example, the above picture is an unfolded cube. The correct answer is the last one.

This list of such good logic questions can be continued indefinitely. All of them share the same common property that they are deterministic in the sense that there are no missing parts in the statement of the question and the student can theoretically solve it without leaving anything to chances. And finally, we cannot forget about classical math problems such as velocity-time-road, percentage of mixtures, work-time-workers, age of people, basic arithmetic etc. The use of these well-known and accepted problem types is still very suitable for IQ tests.

At the end, I recommend having a state commission consisted of mathematicians (preferably experts in mathematical logic and foundations of mathematics) and educators to monitor the quality of the logic questions proposed in important exams. This commission can also determine the standards for now completely unregulated field and ban some of the questions types that are prevalent today. I must also note that I am probably not the first person who is uncomfortable with these problems. I was not surprised to learn that this year some of the elite lyceums in Azerbaijan refused to use them in the entrance exams. I should also note that in general the logic questions are not something against mathematics and they are trying to promote mathematics in its simple everyday format. Cleverly constructed logic questions, in my opinion, are forming a new genre in recreational mathematics and can be successfully used in the future to measure educational achievement but it should be done under constant supervision of experts including professional educators and interested mathematicians.

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