Improvement of mathematical tests for admission examination and its influence on an estimation

Nizami A.Kerimli, Modern Educational Complex
Tofik M.Rasulov, Azerbaijan State Oil Academy

Contents

Selection of the items offered on entrance examinations pursues two interconnected purposes:

1. Knowledge certification of high schools graduates
2. A choice of entrants for the bachelor degree.

Thus, offered items on the mathematics according to the curriculum should reveal a level of knowledge and skills of high schools graduates and also to help with their further studying in universities and becoming competent experts in the area.

Therefore in the presentation the big attention is given on three basic questions:

1. Requirements for the tests
2. Preparation of the items revealing real abilities of applicants.
3. The statistical analysis of positive influence of improving items for progress of students (based on an example of first rate students of Azerbaijan State Oil Academy for last 5 years).

1. Requirements for the tests

In secondary education system basically *pedagogical testing* is used [1]. Each item of such tests allows to define a concrete quality of the applicant, and as a whole testing estimate not only a level of theoretical knowledge, but also skills of its application, allowing objectively to estimate successes in the field of education [5,6,7].

The items included by State Students Admission Commission (SSAC) in entrance examinations are basically **"tests of achievements"**, i.e. they should place high school graduates on a level of their knowledge. Thus the purpose of this testing is to **define a rating of applicants** and to distinguish them from each other.

Alongside with it testing should check up, whether a level of the chosen applicants corresponds to the necessary requirements or not. The answer to this question is given with **"tests of quality"**, directed on an **estimation of a concrete qualities of applicants**.

The basic attention in **"tests of achievement"** is given not so much to the items allowing to estimate applicant's concrete knowledge, as to the items which are fulfilled well by more prepared and are fulfilled discontentedly by poorly prepared applicants.

Actually, the admission examinations spent by SSAC for some late years, beside their direct functions, carry out one more very important function. They estimate the achievement of high schools graduates and thus, as a whole, objectively estimate the work of all secondary education system. Therefore implementation of these duties is not possible only with the **"tests of achievement"** help. The estimation of applicant's concrete knowledge, quality and skills is realized with the help of the **"tests of quality"**.

"Tests of quality" on the mathematics basically should enable to reveal applicant's knowledge of the curriculum i.e. skills of applicants:

- *To arrange the calculations with real numbers, to create algorithms for mathematical solving of real life problems, to use mathematical concepts and formulas;*
- *To arrange identical transformations under rational, irrational, logarithmic and trigonometrical expressions and to estimate corresponding numerical expressions;*
- *To solve the various text problems constructed on mathematical models;*
- *To solve linear, square, fractional-rational, irrational, logarithmic, trigonometrical other equations, an inequalities and their systems,*
- *To define functional dependence between quantities, to apply it to the concrete problems solving, to define direct and inverse dependences, to explain properties of linear, square-law, radical, power, fractional-rational, exponential, logarithmic and trigonometrical functions (the domain, the range, the intervals of increase and decrease, the critical points, etc.), to graph the functions and to analyze them;*
- *To find a derivative of function, to know its physical and geometrical meaning, to apply a derivative for researching of function and solving some problems on an extremum;*
- *To find antiderivative, to calculate the definite integral and to apply it for calculation of the areas and for solving other problems;*
- *To solve the equations, inequalities and their systems containing parameter, and to Investigate dependence of the solution on parameter;*
- *To apply various methods for solving the equations of the mixed type, inequalities*

and their systems;

- *To know sequence concept, to solve problems on arithmetic and geometrical progressions;*
- *To know the basis of the set theory of and to apply elements of combination theory for solving simple probability problems;*
- *To arrange operations with complex numbers;*
- *To make a simple geometrical constructions and measurements (the length, the area the volume), to know other geometrical concepts, axioms and theorems, to use properties of geometrical figures for solving specific problems;*
- *To arrange operations with vectors and to apply vectors for solving different practical problems;*
- *To apply a properties of simple skew fields for solving specific problems and calculating their separate parameters.*

On the other hand, in parallel with reviling of these qualities, mathematical items should reveal the applicant's ability to logic thinking, ability to spend generalization, to make the analysis and conclusions, to model and to build algorithms, to reveal how many other mathematical knowledge and skills are acquired. It is also necessary for the young men further tutoring and work on a specialty, in order they could apply the received knowledge not only in habitual, but also in a new, changed conditions [2, 3].

The important feature of **"tests of quality"** is that by the passage of time their maintenance does not almost change. Such items remind pupils and teachers about the purpose of tutoring and specify what kind of necessary knowledge and skills the applicant should have, according to the curriculum of high school. The negative feature of **"tests of quality"** is that they have low differentiation of quality.

Thus, the chosen items on the mathematics, offered on entrance examinations, should comprise not only the features of **"tests of quality"** which estimate concrete quality, but also the features of **"tests of achievement"**, which allow to arrange precise differentiation. In other words, **items on the mathematics, offered on entrance examinations, should be "tests of achievement", comprising elements of "tests of quality"**.

2. Preparation of the items

Transformation of a usual question into the item is a creative work, demanding the serious efforts. Drawing up the item passes some stages.

In the beginning are defined:

- *The address of the* item *(to whom the* item *is intended).*
- *The purpose of the* item *(what kind of qualities the* item *should estimate).*
- *A level of complexity (easy, medium and difficult).*
- *A category of relevance (the* item *are divided on terminological, factological, generalizing, explanatory, computing, predicting and others).*

At a following stage are defined:

- *Conformity of the item to the curriculum.*
- *Scientific character (uniqueness at the necessary level).*
- *Methodical correctness of drawing up.*
- *Technical parameters (the text should be compact, laconic, demanding small time for the solving, the distracters should be correctly chosen, it should have a low background level stirring to estimate the quality, etc.).*
- *Validness (i.e. ability of the item to revealing concrete qualities).*
- *Differentiation of knowledge level.*
- *Reliability degree.*

Then at the third *pretests* stage, all listed features are examined and respective changes are done. Only after, the items are offered on entrance examinations. After the examinations, each item is studied again in view of entrant's answers and whether it is necessary, amendments are done in it.

First of all, we have to concern some questions with the maintenance of the items. It is known, that "*The equations and their systems*" take very important place in the mathematics and cover some sections (linear and quadratics equations, the equations of the higher orders, fractional-rational, irrational, exponential, logarithmic equations, etc.). Some equations are included in tests of entrance examinations every year. However, it was impossible to receive the information about skills of applicants to solve these items, because by substituting the numbers given in answers in the equations and by checking them, it was possible to find a solution of the equations in many items of such type. Therefore, in the items connected with the equation's solving, questions have been changed and put as follows:

"Find the sum (a difference, product, quotient, etc.) **of the roots of equation … "**
(if there are two roots), or

"What interval possesses the root of equation … "
(if there is one root), or

"Find the number of roots of the equation … "
(if there are many roots connected with the range).

In some cases at solving the equations, change of the form of a question and partially the text, serves also other purpose [4]. For example, we simply wish to estimate skill of the applicant to solve a quadratic equation and so we put the question as follows:

1. Solve the equation $7x^2 + x - 8 = 0$**.**

A) $-1; \frac{8}{7}$ ***B)*** $-8; \frac{1}{7}$ ***C)*** $7; -\frac{1}{8}$ ***D)*** $1; -\frac{8}{7}$ ***E)*** $8; -\frac{1}{7}$

The factors are picked up in this item so, that to solve this equation is much easier, than to substitute the given numbers and to check up. The diagram of this item's work is given on Fig.1. The total relative scores gained by applicants are noted on abscissas, and the percentage of applicants chosen right answers and distracters (*A, B, C, D, E*), and also wrong answers (sign []) are noted on ordinates of the diagram.

Discrimination coefficient:	0.747
Coefficient of correlation with the total score:	0.577
Contribution to reliability:	0.005
The key: *D*	

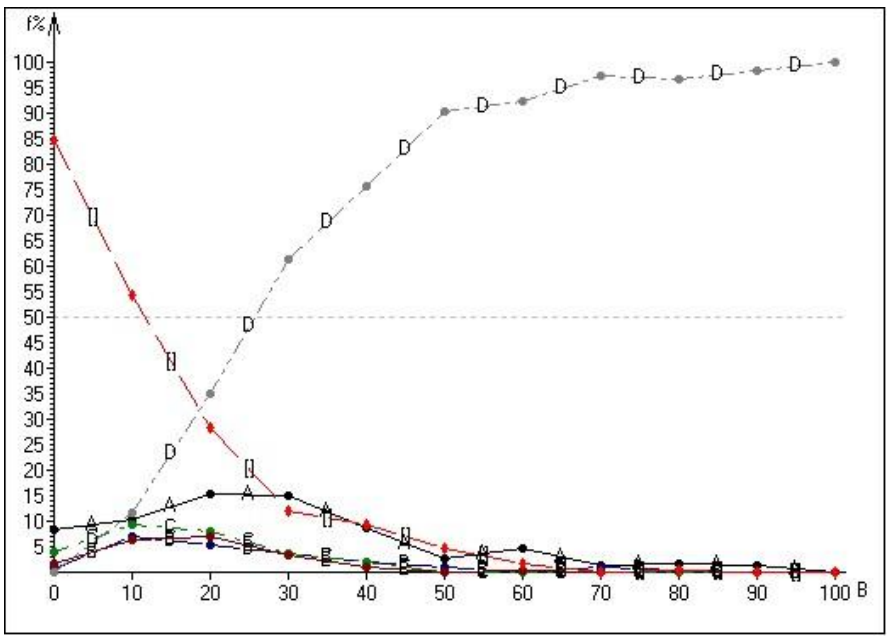


Fig. 1

We can see a very high differentiation ability of this item on the diagram. There was also a high total score of entrants and correlation, and poorly prepared applicants have preferred distracter ***A***.

If we wish to estimate how applicants apply Vièt's theorem, at first we choose factors of a quadratic equation so that the finding of its roots by the known methods would be difficult and the received roots would be irrational, and secondly that the question corresponded to the Vièt's theorem. Let's consider, for example, following task.

2. Calculate value of expression $x_1 \cdot x_2 \cdot (x_1 + x_2)$***,***
where x_1 ***and*** x_2 ***is the solutions of the equation*** $x^2 + 5x - 3 = 0$***.***

A) – 15 ***B)*** 10 ***C)*** 8 ***D)*** 12 ***E)*** 15

Under Vièt's theorem the answer of the given item is obtained very quickly:

$$x_1 \cdot x_2 \cdot (x_1 + x_2) = -3 \cdot (-5) = 15$$

There is no necessity to make comments, what kinds of difficulties at the solving of this task expect the applicant who doesn't know Vièt's theorem. Figure 2 evidently shows that the work of this item completely corresponded to predicted results.

Discrimination coefficient:	0.841
Coefficient of correlation with the total score:	0.660
Contribution to reliability:	0.005
The Key: *E*	

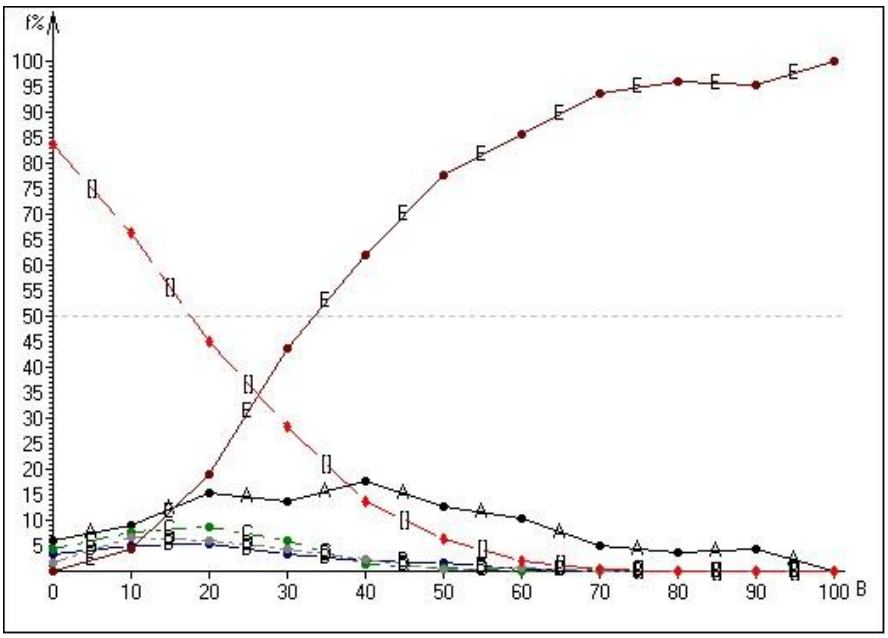


Fig. 2

Practice shows, that in some cases there are problems with wrong understanding of the asked question. For example, in the item connected with a choice of true statement, the sentence:

"Equality $x^2 = 4$ ***is true at*** $x = 2$ ***"***

is often understood doubly. Some people wrongly accept the sentence $x^2 = 4$ at $x = 2$**,** though $x = -2$ satisfies this equality to. Others, simply from $2^2 = 4$ accept it as the true statement. Therefore, the question should be asked so that its understanding

did not depend on various people language perception. For example, in the given item depending on the purpose (i.e. how we wish to give the true or incorrect statement), it would be possible to give the statement like follows:

"Equality $x^2 = 4$ ***is true only at*** $x = 2$***"*** is not a true statement.

"The number $x = 2$ ***satisfies the equality*** $x^2 = 4$***"*** is a true statement.

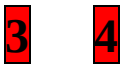
By the way, statement of a question in the form of ***"How many of the statements are true?"*** seems to us not expedient because its qualitative measurement is low. We shall explain our ideas. Let two of five, for example I and II, offered statements are true, and the others are not.

The Key: Two

1. II & IV

2. I & V

3. III & IV



Hence the answer of this item is ***"two"***. Probably some applicants will choose not 1 and 2, but for example 2 and 4, or 1 and 5, or 3 and 4, etc., that is accepting as a true - combinations of two other statements. In this case, the item will not correctly estimate knowledge. But if the question is put as ***"Which of the statements are truly?"*** this problem will be solved. For example,

3. Which statement is true for $a < 0, b > 0$

I. $\frac{a}{b+1} > 0$ ***II.*** $7ab - 1 > 0$ ***III.*** $\frac{b-3a}{7+b} > 0$***?***

A) *only I* ***B)*** *only II* ***C)*** *only III* ***D)*** *II and III* ***E)*** *I and II*

And these item's results also correspond to the predicted results (Fig.3).

Discrimination coefficient: 0.811
Coefficient of correlation with the total score: 0.626
Contribution to reliability: 0.006
The Key: *C*

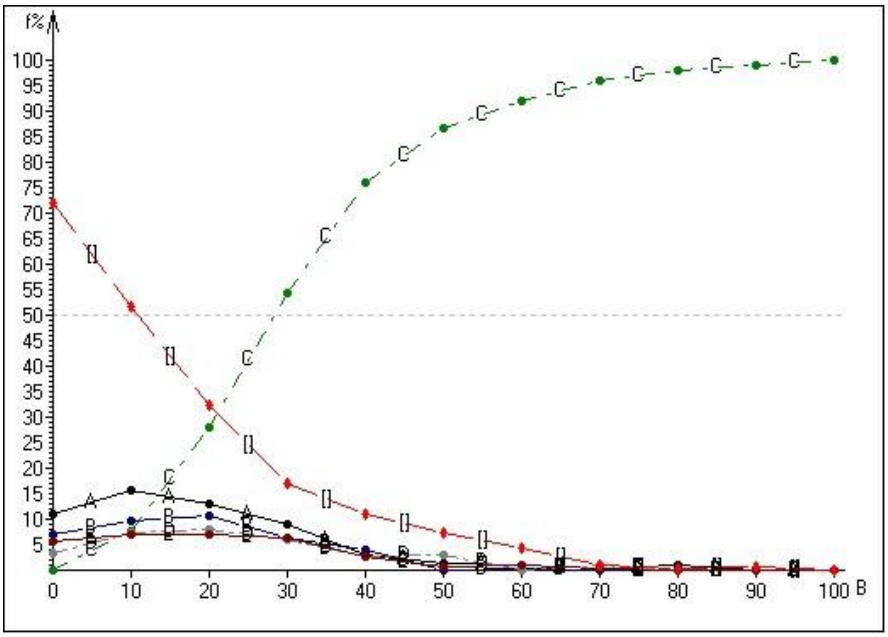


Fig. 3

Statistics of this item on numerical inequalities of average difficulty corresponds to predicted data. The curve corresponding to the key ***C*** increases and the curve of refusing accordingly decreases. The discrimination coefficient, high scores of applicants, correlation coefficient, and coefficient of reliability, being quality indicators of this item are very high.

A choice of distracters also plays a very important role. Let's consider, for example, such item:

4. For the function $f(x)=3x^2-4x+1$ ***find antidirivative which graph passes through the point (-1; 5)?***

A) $F(x)=x^3-4x^2+x+5$ ***B)*** $F(x)=3x^3-4x^2$ ***C)*** $F(x)=x^3-4x^2+x$
D) $F(x)=x^3-2x^2+x+5$ ***E)*** $F(x)=x^3-2x^2+x+9$

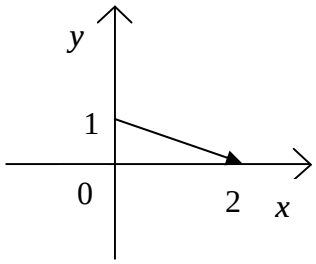
In this item distracters are picked up with a defect, because for finding the right answer it is enough to check up which of given functions passes through the point $(-1;\,5)$. Thus, the answer of this question can be found, having no knowledge about antidirivative. Therefore, this item does not answer the put purpose, i.e. does not allow estimating how many of the applicants have mastered, or have not mastered section "antidirivative function". Having replaced distracters ***B*** and ***C*** with

B) $F(x)=x^3-4x^2+12$ and ***C)*** $F(x)=x^3-4x^2+x+11$

this defect can be eliminated.

One more important problem is connected with correct addressing of the item. For example, we shall consider the simple item, intended to estimate how the applicants can apply the vector coordinates concept.

5. Find coordinates of a vector $\vec{a}$ on a coordinate plane.



A) (2; 1) ***B)*** (1; 2) ***C)*** (–1; 2) ***D)*** (–2; 1) ***E)*** (2; –1)

This simple enough item was examined in humanitarian group and, how to be spoken, "has not worked" (Fig.4).

Discrimination coefficient: -0.025
Coefficient of correlation with the total score: -0.045
Contribution to reliability: -0.004
The key: *E*

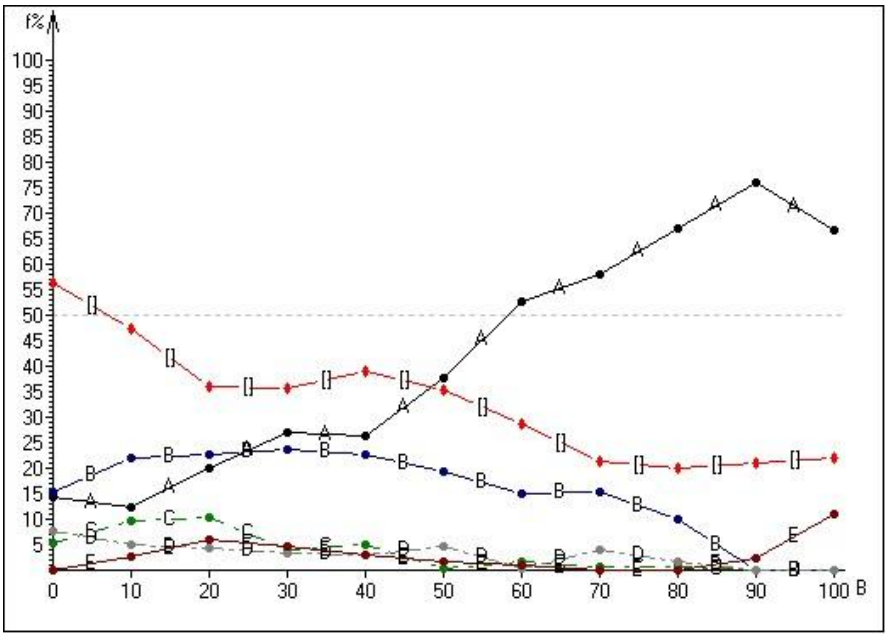


Fig. 4

Apparently, from the Fig.4 the majority of applicants, not having understood the gruff, have noted as a right answer distracters ***A*** and ***B***. Then, this item have been offered to the applicants prepared on economic and technical specialties (on preparatory courses at SSAC, and pupils of the Modern Educational Complex named after Heydar Aliyev) and results of the test corresponded to predicted results.

3. Analysis of the received results

The advantage is given to the items, which are able to revile a better estimation of applicants at last years. There is no doubt that it will lead hereto more worthy entrants have acquired the right to continue their study in universities. The selection of applicants is closely connected as well with their further tutoring. It is necessary to note, that a selection of items on the mathematics, offered on entrance examinations, is spent so, that the knowledge of entrants, got during preparation, helped them at the further studying. And it in turn, has a positive influence on the student's progress in their study in universities.

If during the last years there were some problems in students mastering of separate mathematics sections (such as trigonometry, the limits theory, derivative of a function and its application, antiderivative, definite integral and its applications, a vector and as a whole all geometry), now mastering of these sections was facilitated and it demands less time.

The analysis of examinations results for the last 5 years held by SSAC among students of first rate shows, that the level of students' knowledge has risen. It is shown on the following diagrams. Results of the students who have received positive estimations on "Higher mathematics" course are shown on the diagrams on Fig.5 (autumn semester) and Fig.6 (spring semester), and results of examinations on which students have received only A, and A or B estimations are shown, accordingly, on diagrams on Fig.7 and Fig.8.

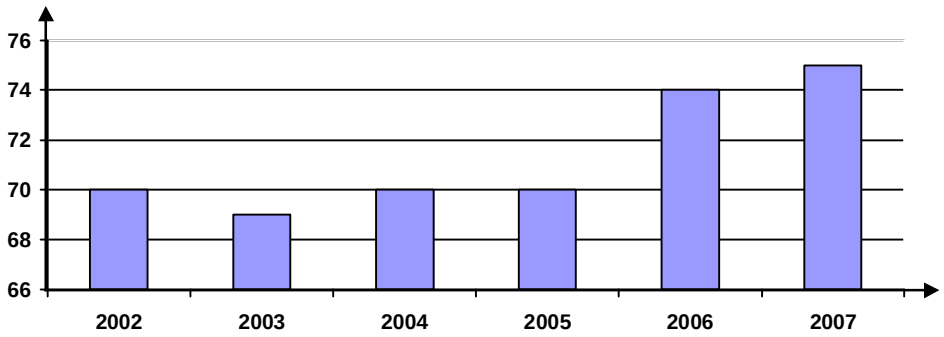


Fig. 5

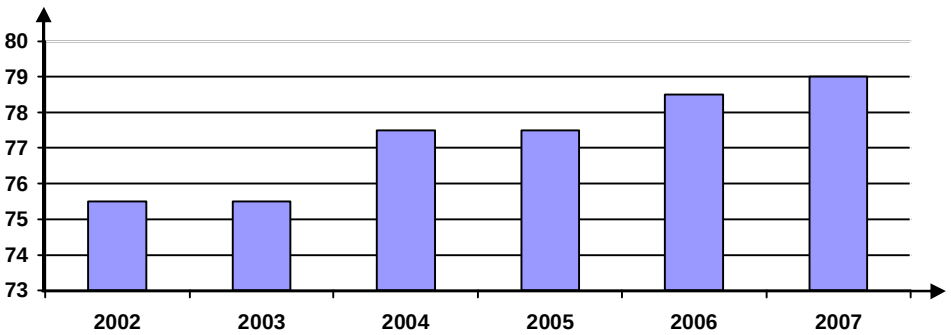


Fig. 6

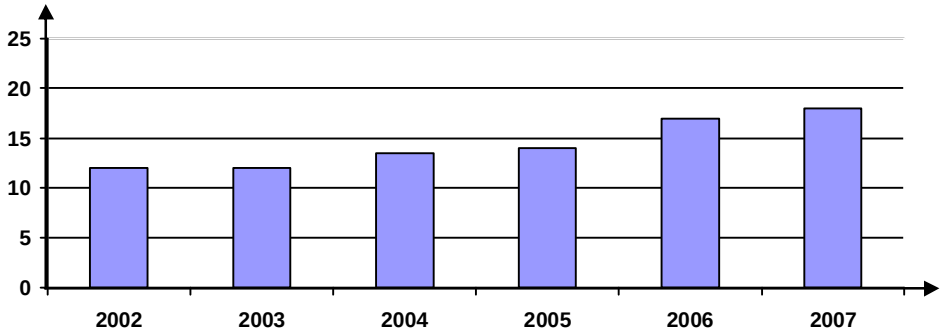


Fig.7

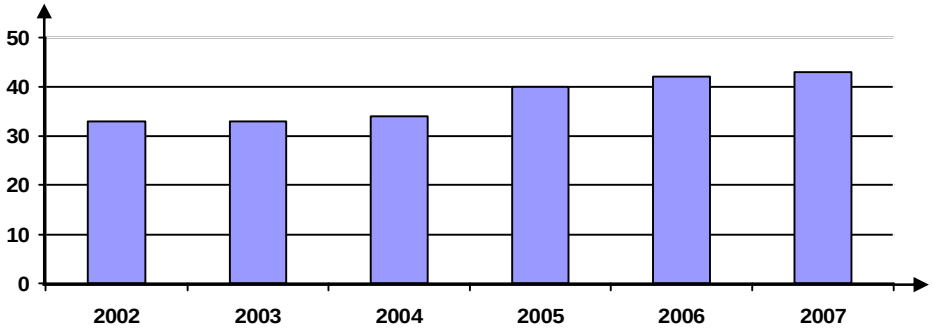


Fig. 8

Thus, it is obvious that improvement of the items on the mathematics creates a basis of more objective choice of students on the step of bachelor and their further successful training. Therefore, it is necessary to continue the scientific and statistical analysis of the items offered on entrance examinations and work on improvement of technology of their preparation and the selection, in order to reach more perfect measuring parameters of the items.

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