Mathematics, language and tests: conflicts and challenges

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Governmental and commercial assessment of mathematics learning across school systems is mediated through language. In particular accessing mathematical ideas requires some use of technical vocabulary. A significant equity problem can arise when definitions of technical terms vary for different groups of students.

This paper looks at the extent to which many common mathematical words in English are given multiple definitions in published dictionaries, and curriculum documents. A particular focus is given to curriculum documents in Australia with additional examples from England and South Africa. Also discussed are strategies employed by test developers to reduce equity issues generated by inconsistent terminology.

To survey the extent to which curricula provide a common set of definitions and meanings 6 documents were examined. Three documents were the glossaries of the curriculum documents of three Australian states. The other three were curriculum glossaries from England, New Zealand and South Africa (English language version). The purpose of this task was not to critique or review the quality of the glossaries (all of which had notable strengths) but to allow for comparisons of definitions.

The documents are referred to in this paper using the following codes:

- NSWK-6: A glossary that accompanies the mathematics syllabus for Kindergarten to Year 6 in the Australian State of NSW (Board of Studies, 2006).
- VIC VELS: A glossary that accompanies the Essential Learning Standards for the Australian State of Victoria (Victorian Curriculum and Assessment Authority, 2007).
- WADraft: A glossary that accompanied the draft consultation on the revised curriculum for the Australian State of Western Australia. (Department of Education and Training, 2007) [Note: this was a consultation document only].
- UKNC: A glossary developed to accompany the English National Curriculum and the National Numeracy Strategy (Qualifications and Curriculum Authority, 2003).
- NZ: A glossary that accompanies the New Zealand curriculum document for mathematics (Ministry of Education, 1995).
- SouthAfrica: A glossary that accompanies the mathematics national curriculum document (English language version) for South Africa (Department of Education, 2002)

It should be noted that some of these documents are no longer current or accompany curricula that are under revision.

Each word, term or phrase was added to a database along with the definition provided. To allow for comparison a further field was added where a standardised version of the term was included – to allow for small variations in spelling, hyphenation or plural/singular usage. For example these terms appear in all 6 documents:

NSWK-6	Stem-and-leaf plot
NZ	Stem-and-leaf graph
SouthAfrica	stem-and-leaf display
UKNC	stem-and-leaf diagram
VIC VELS	stem-plot
WADraft	Stem-and-leaf plot

Each term was flagged by the term “stem-and-leaf” in the database. A cross tabulate table was then produced from the database showing which states defined which terms and how many states defined a particular word.

The table below shows the eleven common entries for each document.

Count of word	Syllabus						
word	NSWK-6	NZ	South Africa	UKNC	VIC VELS	WADraft	Grand Total
vertex	1	1	1	1	1	1	6
stem-and-leaf	1	1	1	1	1	1	6
pyramid	1	1	1	1	1	1	6
prism	1	1	1	1	1	1	6
polyhedron	1	1	1	1	1	1	6
polygon	1	1	1	1	1	1	6
perimeter	1	1	1	1	1	1	6
mode	1	1	1	1	1	1	6
median	1	1	1	1	1	1	6
mean	1	1	1	1	1	1	6
continuous data	1	1	1	1	1	1	6

Surprisingly the words all these glossaries have in common is 11 words. Those words are:

- continuous data
- mean
- median
- mode
- perimeter
- polygon
- polyhedron
- prism

- pyramid
- stem-and-leaf (diagram)
- vertex

Looking only at the three Australian documents the total rises to 18. There are 10 words defined in five documents that aren't defined in the NSWK-6 glossary.

- angle
- congruent
- edge
- equation
- face
- net
- ratio
- rational number
- reflection
- rotation

Arguably 'equation', 'ratio' and 'rational number' are missing because of the age scope of the document. A term like "angle" may be so basic that definition is not required just as it would seem superfluous to define words such as "one" or "two".

There are 29 words in the NSW glossary that don't appear in the other 5. Further some terms are explained within the syllabus but not included in the glossary (e.g. product and the names of some quadrilaterals).

The second stage was to identify from the literature words or terms that have problematic qualities regarding their meaning.

Rothery (Rothery, 1984) classifies word within Mathematical English (ME) or the mathematics register (Pimm, 1987) into three categories:

1. Words which have the same meaning in ME as in OE
2. Words which have a meaning only in ME
3. Words which occur in both OE and ME, but which have a different meaning in ME from their meaning in OE

(where "OE" means Ordinary English). A different paraphrasing of the same idea can be found in Dickson (1984) and a more extensive set of categories is given in Thompson & Rubenstein (2000).

All words in category 3 can be regarded as problematic in an assessment situation in so far as a student may read a word as if its meaning where the Ordinary English meaning rather than the meaning from the mathematical register. Examples would be were students were asked to find the difference between two numbers [(Matthews, 1980) cited in Dickson (1984))] and respond by describing features which differ (one is big, the other small etc) rather than subtracting. Words in this category from Rothery (1984) include: difference, product, parallel, odd, mean, value, similar, circular, divide, average, reflection, remainder.

Pimm (1987) identifies the following words as 'borrowings' and some may fall into category one rather than three: face, degree, relation, power, radical, complete, integrate, legs, product, moment, mean, real, imaginary, rational and natural. Not all of these are necessarily problematic in so far as adding an ambiguity of meaning. However the term 'diagonal' with a meaning of 'oblique' in OE is identified as problematic. Pimm refers to this as “semantic contamination” were the common meaning is used to attempt to make sense of a specialist usage.

An additional issue identified by Pimm (1987) is that some words that we may classify as 1 have meanings in the mathematics register that extend their ordinary meaning. As such they may be better classified as category 3. Examples include enlargement (in mathematics an enlargement can make an object smaller), fraction (which in ordinary English suggests some smaller part but which can be mathematically large). A case identified by Hersh (1997) in describing “math lingo” is “number” which in ordinary English may not always include 0 or even 1 (as in “a number of things” suggests a plurality)

Category 2 words may be problematic in an assessment context primarily in so far as the student simply does not know the word. A case cited in Carpenter et al (1981) of forms of an item from the National Assessment of Educational Progress. In a form where the term “perimeter” is used (arguably a category 3 term) % correct figures were substantially lower than were the phrase “distance all the way around” was used instead. [cited in (Dickson, 1984)]. Additionally Ellerton et al (2000)and others have noted an additional issue where specific maths vocabulary may have inconsistent definitions. The most notable example is the word “trapezium” which refers to a shape with no parallel sides in US English and in British and Australian English refers to a shape with either exactly one pair of parallel sides or at least one pair of parallel sides.

Ellerton et al (2000) goes on to discuss the issue of shape terminology in relation to Van Hiele levels of understanding of geometrical concepts (Mason, 1998). To some extent an awareness of the inclusivity of some geometric properties relates also to the definitions of geometrical terms. For example understanding that an equilateral triangle is also an example of an isosceles triangle relates to knowing that definition of an isosceles triangle is that it has at least two congruent sides (rather than exactly two congruent sides).

In addition to words identified in the literature I have also maintained a list of words that have generated inquiries either in the review of items in formal tests or from teachers, students and parents in relation to test items.

Finally the database was examined to look for two kinds of conflict:

- Concepts with different names (e.g. “sector (pie) graph” NSW K-6 glossary versus “pie graph” NZ)
- Names with different definitions (e.g. “cross-section” in NSW is a plane that cuts an object “parallel to the base”, while in the Victorian VELS glossary this condition is not included). These terms when then compared against specialist Mathematics Dictionaries.

What follows is a list of words that I regard as being problematic in a test context. The list is not exhaustive and is confined to approximately an 8- 14 age range.

Shapes

- Trapezium: has three distinct meanings in English. No parallel sides (US English c.f. TIMMS release item M041300A3 (International Association for the Evaluation of Educational Achievement, 2007)), exactly 1 parallel side (UKNC and SA), at least one parallel side (NSW K6)
- Square, rectangle, oblong: definitions of square and rectangle typically consistent with the inclusive meaning but confusion is common. The non-inclusive term 'oblong' is arguably helpful (Pimm 1987) but is sometimes poorly defined c.f. “oblong: another word for a rectangle or rectangular” (Illustrated Maths Dictionary 3rd Edition (DeKlerk, 1999)). Of the glossaries listed only UKNC defined “oblong”.
- Isosceles triangle: the inclusive nature of the definition is not always clear (e.g. NSW K6 “A triangle with two sides equal in length” can be read two ways – exactly two or at least two).
- Kite: inclusivity of a rhombus or square is also an issue for kites e.g. SA “a quadrilateral with two pairs of adjacent sides equal in length, but not all four sides the same length” compared with UKNC “A quadrilateral that has two pairs of congruent adjacent sides.” The term is not defined in three of the glossaries studied. The NSW K6 curriculum gives a Venn diagram of quadrilaterals that does not include kites, possibly because their relation to other quadrilaterals is hard to show in a Venn diagram. Origo Handbook (Origo Education, 2007) gives rhombus as a special case of a kite.
- Quadrilateral: has an additional more extended meaning to include cross quadrilaterals such as bow-tie shapes which may not be regarded as legitimate. (Weisstein)
- Polygon: same issue as quadrilateral.
- Prism, pyramids, cones, cylinders: some confusion in some syllabus documents and text books as to whether cylinders are a species of prism (they aren’t) and whether cones are a species of pyramid. The Australian National Mathematics Profiles included the learning objective: “Identify prisms and pyramids (including cones and cylinders)”. The NZ glossary says that prisms are a “polyhedron of

uniform cross section with two congruent and parallel end faces" but also says that a cylinder is "A prism of circular cross section."

- Circle: well defined but a more casual use appears in some test instructions as a mode of response (e.g. TIMMS release item M041056 which asks that students "Draw a circle around 1/3 of the cookies." An actual circle can't really be drawn.
- Cross-section: Either a synonym for "section" (VIC Vels) or as a specific kind of section perpendicular to an axis (or equivalently parallel to a face). Origo Maths Handbook (Origo Education, 2007) gives the second definition.
- Enlargement: either the general mathematical sense (which can include a reduction in size) or a restricted sense which conforms to Ordinary English usage (e.g. VIC Vels "One shape is an enlargement of another shape if they are similar and the scale factor for dilation is greater than 1."
- Net: unclear as to whether only polyhedra can have nets or whether shapes with curved surfaces can also have nets (e.g. a net of a cone) WADraft "A two-dimensional plan which can be used to make a three-dimensional shape." Contrasts with "A pattern of polygons that can be folded to form (the surface of) a polyhedron." NZ

Numbers

- Number sentence: a term widely used but largely undefined. (see TIMMS release item M041281 (International Association for the Evaluation of Educational Achievement, 2007))
- Decimal: technically a number in Base 10 but used as a shorthand for a decimal fraction. Decimal fraction is in turn problematic as "fraction" is often used as a shorthand for "number written in fraction notation" e.g. "records a remainder as a fraction or decimal, where appropriate e.g. 25 ÷ 4 = 6 or 6.25" NS3.3 page 161
- Fraction: can be used as a synonym for a rational number, as a particular way of representing rational numbers and also as a synonym for proportion. (see TIMMS release item M022043 (International Association for the Evaluation of Educational Achievement, 2007)) Some glossaries distinguish between "fraction" and "fractional number" (e.g. NZ equivalent fraction is defined as "Fractions that name the same fractional number")
- Power: either the answer you get after exponentiation but sometimes used to refer to the index ("to the power of 3").
- Whole number: typically the positive integers and zero but sometimes as a synonym for counting number or for integer
- Natural numbers: similar to whole number, sometimes includes zero (see VIC Vels) and sometimes does not (e.g. NZ and UKNC)
- Even/odd number: is zero an even number? Is -4 an even number? An unclear definition of whole number can lead to further issues with subsets of whole numbers. UKNC defines "even number" as "An integer that is divisible by 2."

- Composite number: As above sometimes confined only to positive integers greater than 0 (e.g. NZ and VIC VELS) and sometimes given more generally (e.g. “A number that has more than two factors” NSW K6).
- Number: in Standard English a number of things means that there is more than 1.

Data

- Average: as a synonym for mean or as a generic term for measures of central tendency.
- Mean: generally has a restricted meaning in school level mathematics to the arithmetic mean
- Pie chart: the term “sector graph” is almost wholly confined to NSW maths teachers. In the syllabus it is now the chimerical sector (pie) graph.
- Bar chart: The NSW syllabus explicitly states “ the term ‘bar graph’ is reserved for divided bar graphs and should not be used for a column graph with horizontal bars” DS1.1 page 86 (Board of Studies, 2006). This pronouncement doesn’t match either common usage or the usage in other syllabus documents.
- Assorted other statistical diagrams: scatter plots have assorted names, box-and-whisker are alternatively graphs or plots or box-plots. Only two of the glossaries gave stem-and-leaf graphs the same name.

NSWK-6	Stem-and-leaf plot
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Algebra

- Pronumeral: a term still in use in Australia but rarely defined. For example Vic Vels glossary uses pro-numeral in definitions “Equations are used to assign a value to a pro-numeral” but doesn’t define “pro-numeral”

Others

- Length, width, height, depth, breadth: these terms are sometimes used synonymously and at other times there are apparent conventions on how we apply them to specific objects
- Diagonal: properly a straight line joining two non-adjacent vertices but has a non-mathematical usage equivalent to the word “oblique” and a definition by extension from numerous quadrilateral examples of a line that joins two

OPPOSITE vertices. For example the SA glossary gives this definition: “line joining one vertex of a polygon to an opposite vertex”

Discussion

The meaning of mathematical terminology in a classroom can be problematic. However communication between teacher and student (or peers) is dynamic and two-way. Misconceptions may go undetected but once observed than they can be explained.

Communication in formal assessment, particularly in the traditional pencil-and-paper test, is quite different. Meanings cannot be clarified without adding more words – a self limiting process as this in turn increases reading load. The student is not always in a position to ask for further clarification of meaning. Typically the communication from the student is carefully restricted to forms of response that can be marked systematically.

When presented with an item with mathematical terminology there are a number of possibilities that could generate errors:

1. the student does not know the term at all
2. the student has some familiarity but is unsure of its meaning
3. the student use a meaning from the wrong register (an ordinary English meaning of “difference” for example)
4. the student has misconception of the meaning of the term (for example considering squares as not being rectangles)
5. the student has been taught a similar but different term for the same concept and is consequently confused (is a square pyramid the same as a square-based pyramid?)
6. the student has been taught a different meaning for the term (narrower or broader)

The exact construct being tested has some bearing on the extent to which these errors are problematic for the equity and validity of the test. In some tests of a specific curriculum where knowledge of these terms is explicitly regarded as part of the construct points 1 to 4 may be regarded as errors by the student that are relevant to measuring the construct. In a test whose purpose was to evaluate the teaching of a specific curriculum all 6 errors might be regarded as legitimate points to test.

Test were the construct is less closely tied to a curriculum all these points might be regarded as errors independent of the construct to some extent. However even in a test whose construct was a more abstract notion of mathematical ability total avoidance of mathematical terminology might not be practical (or may even be counter-productive requiring additional wording to explain concepts that could be neatly encapsulated with a technical term).

Reducing potential problems in assessment caused by such errors could be done in a number of ways:

- Using forms of assessment that allow for more flexible communication such as interviews or rich tasks.
- Avoiding the use of problematic technical terms in test items completely.
- Using only technical vocabulary that is several stages below the target level of the test. For example using Year 6 vocabulary in a Year 9 test. This in itself may be problematic.
- Providing a glossary of technical terms in advance of the test to establish common meanings.
- Allowing students access to a glossary of technical terms during the test.
- Accommodating non-standard definitions in mark schemes (e.g. allowing a 'bow-tie' shape to be drawn in an item that asks students to draw a quadrilateral).
- Designing items that rely only on common meanings between competing definitions. For example in a question about trapeziums ensuring that for some additional reason parallelograms are excluded.

In each case further research is needed. Avenues of research should include:

- Establishing how familiar students of given year levels are with appropriate technical vocabulary.
- That in turn suggests the need to develop adequate assessments of technical mathematical vocabulary.
- Surveying teachers of mathematics for their understandings of problematic technical terms.

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