Redesigning the CEM Mathematics Diagnostic Tests as Developmental Assessment Instruments

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<u>Abstract</u>

Current efforts of the Center for Educational Measurement to redesign its diagnostic tests in mathematics across six grade levels are guided by the developmental approach to assessment adopted by the Australian Council for Educational Research. This approach generates progress maps which place a learner's skills and knowledge along a typical sequence of development as the learner moves within one grade level and on to the next level. Progress is measured in terms of degree of mastery of content and subsequent attainment of higher levels of performance. This is based on the notion that one's competence in an area of learning improves over time.

The progress map described in this paper is drawn from a synthesis of logical connections among the contents and skills found in the various learning areas covered by the CEM mathematics diagnostic tests. The map is the result of a series of consultations between CEM test developers and subject area experts. The competencies measured by the tests are specified by a national core curriculum – the 2002 Basic Education Curriculum – with the inclusion of some topics not part of the core but found to be commonly taken up by a surveyed sample of private schools. This paper also explores the impact of progress maps on (1) measuring a learner's growth, and (2) aligning assessments to development across the curriculum.

Background

Basic education in the Philippines involves six years of compulsory elementary education and four years of secondary education¹. Filipino students, on the average, finish elementary school at age 12 or 13 years and secondary school at age 16 or 17 years. Afterwards, they may enroll in tertiary institutions to obtain a degree or a certificate in a course of their choice

The Philippines has a national curriculum. The Department of Education (DepEd) prescribes the content areas and learning competencies for all levels. The Department's Bureau of Elementary and Secondary Education defines the specific learning competencies under each of the content areas in all subjects in each grade or year level. This bureau is responsible for developing, publishing, and disseminating these specific learning competencies to the field. The competencies in each learning area are expected to be mastered by the student at the end of each grade and year level and at the end of their elementary and secondary schooling.

It must be noted, however, that schools are given the option to modify the national curriculum to suit local contexts. Variations may be seen in terms of content sequence, teaching strategies, or other co-curricular activities that could further enhance learning. In fact, the DepEd

¹ As of school year 2004-2005, government schools constitute 89% of the total number of schools at the elementary level and 59% at the secondary level. The rest are privately-funded.

does not discourage such modifications as long as the basic requirements of the curriculum are fulfilled (Mariñas and Ditapat, 2000).

Restructuring the Curriculum

Studies made by the DepEd revealed an overloaded basic education curriculum that has been implemented for more than a decade. There were too many subjects and the required competencies and daily assignments for each subject could not be tackled reasonably well within a school year. As a result, the teaching and learning of the "untaught" competencies would be given priority in the following year because they were preconditions to learning of higher-order skills for that year level. Consequently, the DepEd restructured the 1983 Elementary School Curriculum and the 1989 Secondary Education Curriculum and came up with the 2002 Basic Education Curriculum, or the BEC (Iwai, 2003). This "new" curriculum was first implemented at all levels (except Grade 6 and Fourth Year) in all public schools in school year 2002-2003. For private schools, its implementation was made optional (Department of Education, 2002).

Changes in the curriculum significantly affected the existing achievement tests of the Center for Educational Measurement $(CEM)^2$. Thus, the subject area tests had to be realigned to the new curriculum. In the process, we injected the developmental approach adopted by the Australian Council for Educational Research.

Redesigning the CEM diagnostic tests

The CEM tests were designed to reflect the achievement of students as they move through the grade levels. Apart from aligning the tests with the prescribed curriculum, there was no deliberate attempt to connect the expected grade level competencies to student growth. The task of articulating contents and skills across the academic ladder is generally assumed by curriculum developers. The information generated by the tests were confined to mere numerical and descriptive reports about what a student's strengths and weaknesses are in the academic subject and how the student's performance compares with those of other students. Thus, instead of connecting performance progressively across time, scores were interpreted discretely.

In 2004, we began to redesign our subject area achievement tests as developmental assessment instruments. Current efforts to improve the design of our tests follow from the recommendations of the National Council Research 2001 Report and the Developmental Assessment Program of the Australian Council for Educational Research. As Tognolini (2004) pointed out, "...the key feature of the (developmental) model of assessment is that the student's progress or growth in the subject is monitored, in much the same way as a child's physical growth is monitored, along a linear continuum that is referred to as a developmental continuum."

Thus, to make tests more informative and useful, they should not be based solely on a hierarchical array of items measuring various levels of difficulty. Conceptually, the tests should be tied to cognition and learning models, enabling them to "yield richer inferences about student knowledge" (NRC, 2001).

² a non-government, non-stock, nonprofit educational testing and research institution in the Philippines.

The developmental assessment approach is based on the notion that one's competence in an area of learning improves over time. It requires the use of a progress map of learning outcomes as basis for monitoring student progress. This map presents a detailed description of the learning competencies and skills arranged in the order by which they normally develop as the learner moves within one grade level and on to the next level. Progress is measured in terms of the degree of mastery of skills and the subsequent attainment of higher levels of performance (NRC, 2001).

This paper describes the first major step taken by CEM toward redesigning its current Mathematics Diagnostic Tests as development assessment instruments – the creation of a progress map in elementary mathematics. The succeeding steps (which will be the topic of another paper) include [1] the validation of the progress map, [2] the restructuring of the map based on validation results, and [3] the establishment of performance standards.

The CEM Elementary Mathematics Diagnostic Tests

The Math tests for Grades 1 to 6 are designed to measure the strengths and weaknesses of students in Mathematics across the six grade levels. The tests cover content areas included in the BEC and the items are classified according to the specific learning competencies and major content areas they measure. (See Appendix for a description of the development of the test.)

Developing the Progress Map

The progress map was drawn from a synthesis of logical connections among the contents and skills found in the various learning areas covered by the existing CEM mathematics tests. The competencies measured were based on the national core curriculum – the 2002 Basic Education Curriculum – with the inclusion of some learning competencies not part of the core but found to be commonly taken up by a surveyed sample of private schools. The survey had to be done because the implementation of the BEC is optional for private schools.

The test specifications for the elementary mathematics tests served as the basis for the content areas and learning competencies that were included in the progress map. The test specifications were shown to a mathematics curriculum expert for review. The expert was asked to [1] make an aligned presentation of the Mathematics Diagnostic Test sequential skills from Grades I to VI and [2] map and establish the interconnections between and among the different content areas based on the critical skills, vertically (within a particular grade level) and horizontally (across all the grade levels). The resulting map was then shown to other mathematics experts for initial validation.

The interrelationship of the content areas and specific learning competencies presented in the map would help teachers and consequently, the students to experience and to view Mathematics as a cohesive whole.

The mastery of prerequisite skills at a certain level prepares the learner to cope with the demands of more difficult concepts and applications in the succeeding level. As one progresses through the learning experiences of higher levels, more opportunities become available for mastering increasingly more complex concepts and competencies and for optimizing the

development of higher thinking skills, including analysis and synthesis, problem solving and decision making.

This paper illustrates the progress map for the teaching and learning of *Fractions*. Over time, the concept of fractions proved to be one of the most difficult learning area for elementary students. True understanding of this area is essential towards having a more realistic perspective of what is encountered daily in life — that there is always an element of "sharing" (for instance, no one is served a whole [uncut] pizza nor is anyone served and expected to finish a whole pot of soup).

Figure 1 shows a portion of the progress map for elementary mathematics. It presents the vertical and horizontal interconnections of some of the learning competencies under the area of Fractions. To facilitate identification, the learning competencies were coded (1A to 6D). The letters represent the specific learning competencies, while the code numbers stand for the grade level under which the competency is found. Figure 2 shows some sample items that measure competencies from 1A to 3C.

A Description of the Content Area Fractions

An illustration of the development of the lesson on comparing and ordering fractions within the grade level and across grade levels from grade 2 to grade 6 is shown as follows:

Grade 2 – Lesson: Compares Unit Fractions Using Relation Symbols (referring to 2D)

The development of this lesson starts with the use of cutouts of circular or rectangular cardboards (*referring to 2A*). The cardboard is partitioned into 3 equal parts with one part (1/3) marked or shaded. Another cardboard of the same size and shape as the first one is partitioned into 6 parts with one part (1/6) shaded or marked. A third cardboard of the same kind as the first two is cut into 5 equal parts with one part (1/5) also marked or shaded. These three unit fractions add up to their first collection of fractions they learned in grade 1, namely 1/2 and 1/4 (*referring to 1A and 1B*). If the first set of cutouts is taken from a circular region, then the same fractions can be shown using, this time, a rectangular region and then a square region. The lesson progresses through the use of a model like a bamboo stick showing 1/3 of it, 1/6 of it, and 1/5 of it. The use of a set of objects (*referring to 2B*), say 30 marbles grouped into 10's (1/3 of the set), grouped into 5's (1/6 of the set), and grouped into 6's (1/5 of the set).

The pupils should be guided in comparing the fractions using the cutouts. The cutouts showing 1/3, 1/6, and 1/5 of the same circular region can be put side by side. The learners can clearly see that the 1/3 piece is bigger in size than the 1/5 piece which in turn is bigger than the 1/6 piece. The same pattern is observed when the learners compare the fractional pieces using the cutouts from the rectangular and square regions. It is even clearer when the children work on three sticks of the same length. They can notice that 1/3 of the first stick is longer than 1/5 of the second stick which in turn is longer than 1/6 of the last stick. In the use of sets, the learners employ counting. A set of 30 marbles shows 10 marbles for 1/3 of the set, 6 marbles for 1/5 of the set, and 5 marbles for 1/6 of the set. Seeing that 10 > 6 > 5, it is easy to say that 1/3 > 1/5 > 1/6. The comparison may include the fractions 1/2 and 1/4, which are taught in grade 1.

FIGURE 1: PROGRESS MAP ON FRACTIONS





FIGURE 2: SAMPLE ITEMS FOR FRACTIONS

<u>**Grade 3**</u> – Lesson: Orders Fractions Less than One, Equal to One, and Greater than One (*referring to 3C*)

It is already clear for the grade 3 learners that the fractions they learned in grades 1 and 2 were just parts of a whole. Hence, 1/3 piece/cutout from a whole (1) indicates that 1/3 < 1. This should be supported with an illustration or model by putting 1/3 piece of a region and 1 whole piece of the same kind of region (*referring to 3A*). To put emphasis on this concept, use cutouts to compare 1/6 and 1 whole, 1/5 and 1 whole, 1/2 and 1 whole, and 1/4 and 1 whole. Once the concept is already clear, other fractional parts can be taken, like 2/3, 3/4, 2/5, 5/6, and other denominators (*referring to 3B*). Other fractional values may now be introduced like shading all the parts of a whole cut into 3 parts, 5 parts or 6 parts to indicate that the fractions 3/3, 5/5, and 6/6 are equal to 1. Use of realistic approach should now be introduced, like cutting 2 apple pies of the same kind/size into 6 pieces each for a total of 12 pieces in all. If 7 pieces are taken, this indicates the fraction 7/6 wherein 1 whole pie is taken and 1 slice from the second pie is taken for a total of 1 and 1/6. The learners can see that this fraction is greater than 1 (*still referring to 3B*).

Comparison of fractions less than 1, equal to 1, and greater than 1 can be easily understood using the cutouts (*referring to 3C*). Comparing fractions with denominator 6 can be easy for the learners if they work on cutouts of 1/6, 2/6, 3/6, 4/6, and 5/6. From these cutouts, they can see that the 1/6 piece is smaller than the 2/6 piece, the 2/6 piece is smaller than the 3/6 piece, and so on until 6/6, 7/6, 8/6, etc. Fractions of other denominators should also be presented. Since they can already compare unit fractions, they are ready to compare other fractions from different denominators.

<u>**Grade 4**</u> – Lesson: Orders Fraction Written in Different Forms from Least to Greatest and viceversa (*referring to 4G*)

A thorough review of lessons in the previous grade is necessary to check learners' level of mastery (*referring to 4A*). The cutouts used in the previous grade can be used again to introduce the terms proper fraction, improper fraction, and mixed form and to use the concept to generate more proper, improper, and mixed fractions (*referring to 4B*). From these cutouts, learners should be able to associate that proper fractions have numerators less than the denominators (2/6), improper fractions have numerators equal to or greater than the denominators (7/3), and mixed fractions has a whole number and a fraction (2 1/5). The same cutouts can be used to group fractions that are similar and dissimilar. Since the learners can distinguish 1/2 from 1/3 as dissimilar, they should be able to associate 1/2 with 3/6 and 1/3 with 2/6. From this, the dissimilar fractions 1/2 and 1/3 are now made similar using 3/6 and 2/6. Other unit fractions can be done/introduced similarly (*referring to 4C*).

There is a need to practice the learners in determining cutouts of the same size like 1/3 which is as big as 2/6, 1/2 being as big as 2/4 or 3/6, 3/4 being as big as 6/8, and so on. The comparison that the learners do on the cutouts should be the basis for ordering the fractions in order of magnitude, i.e. least to greatest and greatest to least (*referring to 4G*).

After a thorough exposure to cut outs, passing the concrete and semi-concrete stage, and with a deeper understanding of similar and dissimilar fractions (*referring to 4C*), the learners are now ready to tackle changing dissimilar to similar fractions by finding the LCD of a set of fractions (*referring to 4E*). The LCD of two or more fractions is the least common multiple of

the denominators. Such is used to transform dissimilar to similar fractions (*referring to 4F*). At this stage, the learners should be guided in using the basic division and multiplication processes, i.e., when dissimilar fractions like 2/3 and 3/5 are changed to similar fractions, the learners will use 15 as the LCD and this will be divided by the denominator 3 and the quotient will then be multiplied by 2 resulting to 10/15. Hence, using the same method, 3/5 will be transformed to 9/15. With this multi-step process, the learners will visualize then that 2/3 and 3/5 are the same as 10/15 and 9/15. This leads the learners to order fractions written in different forms from least to greatest and vice versa (*referring to 4G*). Hence, the pupils will arrange the given set of fractions in this order: 3/5, 2/3, as the correct sequence.

<u>**Grade 5**</u> – Lesson: Orders Dissimilar Fractions Written in Different Forms from Least to Greatest and vice versa (*referring to 5E*)

After a certain phase of development and acquisition of basic skills namely: identifying fractions involving regions and sets (*referring to 4A*), and identifying the different kinds of fraction (*referring to 4B and C*), the learners are ready to tackle bigger numbers and different forms of fractions. This requirement is essential in enhancing the learners' skills in changing improper fraction to mixed numbers and vice versa (*referring to 5A*). This too, is important in comparing fractions and mixed numbers by using the cross product method (*referring to 5B*).

Comparison of fractions say 4/6 and 3/7, can best be attained by employing the cross products method. With this step, one numerator of a fraction and one denominator of another fraction are multiplied, specifically 4 and 7 and 3 and 6 are paired off to get the cross product. Hence, 4 and 7 gives a product of 28, while 3 and 6, is 18. Since 28 (as placed on top of 4/6 as part of the step in cross multiplication) is bigger than 18 (also as placed on top of 3/7), therefore the learners will say that 4/6 > 3/7. This method can also be enhanced by finding and applying the LCD of two or more fractions (*referring to 4E as a review and to 5C as reinforcement*).

This particular lesson or skill requires a thorough review of a similar lesson given previously in grade 4. Gradually, dissimilar fractions involving denominators with 1 or 2 digits i.e., 5 /6, 2/9, 3/11, 7/10, 5/13, and the like, may now be used to demonstrate how dissimilar fractions vary in shapes (referring to semi-concrete stage). Requisite to this process is the extensive recall of the concept of least common multiple which is essential in finding the least common denominator of two or more fractions (*referring to 5 C*). At this stage, the learners are well equipped with the basic concepts of number theory wherein the crucial concept of LCM is enhanced.

With the initial background in changing dissimilar to similar fractions obtained from Grade 4 (*referring to 4F*), the learners are now ready to explore finding the LCD with at least 2-digit denominator. A smooth take off in finding the LCD of a set of fractions is intensively relevant to changing dissimilar to similar fractions (*referring to 5D*). Through this, the learners will have a clear idea on how to transform fractions like 1/6, 4/15, 2/3 and 7/30 to similar fractions. By procedure, the learners will find the LCM of 6, 15, 3, and 30 which is actually 30. 30 serves now as the least common denominator or the LCD. Through a step-by-step procedure, the learners will divide 30 by the denominator 6, of the first fraction and its quotient, which is 5, will be multiplied by the numerator 1, being the numerator of the first fractions by using the same steps, would yield 8/30, 20/30, 7/30. Hence, when 1/6, 4/15, 2/3 and 7/30 are arranged from least to greatest, this will translate to 1/6, 7/30, 4/15, and 2/3.

With a clear execution of each step based on the example provided, the learners will eventually apply the easiest step to arrange dissimilar fractions from greatest to least or vice versa (*referring to 5E*) with ease and self-fulfillment.

<u>**Grade 6**</u> – Lesson: Orders Fractions in Simple and Mixed Forms in Ascending and Descending Order Using different Methods (*referring to 6D*)

Developmental review of the past lesson is necessary since the lesson is an extension and at the same time expansion of related skills taken in grade 5. To make it challenging for the more mature learners, preparation of a good variety of fractions need to be done to make the activity more challenging and more interesting. Variation of activities from simple to complex is a must for a group of dynamic learners having different needs, interest and motivation.

After a through assessment on the basic concepts of the different kinds of fractions including the skill in converting mixed numbers to improper fractions and vice versa (*referring to 6A*), the learners are expected to manifest mastery in comparing fractions (*referring to 6B*). This particular skill must be properly mastered prior to acquiring the skill in finding the LCD, which have been intensively introduced in Grades 4 and 5. Both are important and necessary to order fractions in simple and mixed forms in ascending or descending order using different methods (*referring to 6D*).

The same procedures used in the previous level will be employed in ordering fractions using different kinds and forms. The main focus in this level is for the learners to acquire proficiency in arranging different fractions in sequential order.

Proposed Validation of the Progress Map

A longitudinal study of Grades 1 to 6 students, using the CEM Mathematics Diagnostic Test results, will be undertaken to verify the sequential development of skills within a particular grade level and across grade levels as described in the progress map. Item responses may be analyzed to determine patterns or trends in the acquisition of skills. Interviews of students and teachers may be conducted to establish confirmatory evidence. Students may be probed on their understanding of concepts and solutions to problems. The teachers may contribute their actual classroom observations on how their students' learning of the subject match the theoretical framework of the progress map.

For the next three years, we have concrete plans to extend the study to secondary level mathematics as well as to other academic subjects in order to complete the series for the basic education curriculum progress maps. A related three-year study on non-cognitive factors that affect mathematics learning is now in progress, where we hope the results could help us explain some of our initial findings from the largely cognitive-based research we have started.

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APPENDIX

Development of the Mathematics Test

The development of the mathematics test starts with the preparation of the test specifications which delimits the achievement area and the behavioral objectives to be tested, the number of questions to be included, and the types of items to be used for each level. These specifications are aligned with the 2002 Basic Education Curriculum of the country's education department. CEM works with subject area experts to review various syllabi, textbooks, and the current curriculum for scope and sequence. After thoroughly discussing the test specifications with the consultant(s), CEM contacts other subject area experts to write test items based on the final specifications.

The consultants who prepare the test specifications for a particular grade level also act as reviewer of the items for the same grade level, and revises them if necessary. The test items are evaluated in terms of format, matching with the specific learning competencies, and accuracy and appropriateness.

Two to three equivalent forms for each grade level measuring the same skill and ability are assembled. These forms are then pretested to determine the appropriateness of the test items for the intended grade level, the statistical characteristics of each item, the clarity of directions and the adequacy of the time allotments.

Pretest results are used in determining acceptability of the items — through the process of item analysis. Items are evaluated based on their statistical characteristics, such as level of difficulty, level of discrimination, and effectiveness of distracters. Test items with good statistical characteristics are

accepted and selected for the final form. Item selection for the final copy of the test is guided by the test specifications to assure balanced coverage of the contents and skills to be measured. The final form is then administered to a sample of schools. The data from this administration is used for establishing norms and in determining test reliability and validity.

Development of Norms

A sampling scheme was used in the selection of the norm groups of the Grades 1 to 6 Mathematics Tests. Schools in the country were classified into high, middle, and low categories based on their performances in previous nationally administered tests. Schools were then selected in order to get the desired representative sample across the three major islands in the country — Luzon, Visayas, and Mindanao. The resulting norm group had a performance profile which approximated a normal distribution: 68% for the middle group and 16 % for both high and low groups, with the school as the basic sampling unit.

During the last two months of school year 2004-2005, the Elementary Mathematics Tests for grades 1 to 6 were administered to representative samples of grades 1 to 6 students from 20 participating elementary schools. The selected samples for norming across the different grade levels had equal proportions of male and female examinees.

Table 1 presents the distribution of the norm sample according to geographic location. Though each grade level test was administered to the same number of schools, the number of examinees varied across the grade levels – with Grade 1 (1,192) having the most number of examinees, closely followed by Grades 2 (1,186), Grade 5 (1,177), and Grade 4 (1,168). Grades 3 and 6 had the least with 1,105 and 1,018, respectively. With regard to geographic representation, the Luzon area, in all grade levels, had the greatest number of examinees while the Visayas area had the least.

	Norm Groups					
Location	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
Luzon	46%	46%	49%	50%	50%	41%
Visayas	20%	21%	21%	19%	21%	24%
Mindanao	33%	33%	30%	31%	29%	35%

 Table 1. Percentage of the Elementary Mathematics Tests Norm Groups According to Geographic Location

Note: The number of students is the unit of distribution used.

Test scores

The Center for Educational Measurement's (CEM) Mathematics Achievement/Diagnostic Tests give both criterion-referenced and norm-referenced information. The criterion-referenced scores generate information on the strengths of students in specific content areas (e.g. Whole Numbers, Fractions, Decimals, etc.) and cognitive skills (e.g. Knowledge, Computation, Comprehension, and Application). The norm-referenced scores, on the other hand, yield information on the comparison of a student or a school's performance with the norm group's performance on the same tests. The scores for each area and skill are expressed in percent correct, while the score on the whole test, called the overall score, is expressed in percent correct, standard score, percentile rank, and quality index.