The Correlation Heuristic: Interpretations of the Pearson Coefficient of Correlation are Optimistically Biased

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Abstract

This research examined the relationship between the Pearson correlation coefficient and the mean absolute differences between paired standard scores. Study 1 compared a theoretically derived relationship between the Pearson correlation values and their corresponding mean absolute differences between paired standard scores, and the corresponding empirical relationship obtained from three databases: self-administered questionnaires, students' entrance scores and grade point averages, and students' course grades. The theoretical and empirical relationships were similar for correlation values ranging between 0 and 0.6, reflecting approximately linear relation, with an intercept value of about 1.1 and a slope of about -0.7. Study 2 examined people's subjective interpretations of this relationship. Most participants, both undergraduate students and more statistically proficient participants, also estimated the relationship to be linear, though, with an intercept of 1 and a slope of -1. These subjective estimations are systematically biased compared to the theoretical and empirical values, underestimating the mean absolute differences between paired standard scores by 0.10-0.35 for correlation values ranging from 0 to 0.6. The paper discusses the negative consequences of these biased interpretations of the Pearson correlation coefficient and offers possible procedures for their correction.

Keywords: Pearson Correlation; biased interpretation of correlation; heuristic thinking

Introduction

The Pearson correlation coefficient (r_p) is a central measure of association between variables, widely used in Social Sciences research. Rodgers and Nicewander (1988) detailed 13 formulations of the Pearson correlation coefficient, and several additional formulations have been offered since (e.g., Rovine and von Eye, 1997). One formulation of r_p, which is less frequently used, is an inverse linear function of the mean square differences between all paired standard scores (Cahan, 2000; Falk and Well, 1997; Rodgers and Nicewander, 1988):

(1) $r_p = 1 – 0.5 * \text{Mean}(D^2)$,

where D is the difference between paired standard scores. Extracting $\text{Mean}(D^2)$ from Formula 1 results in:

(2) $\text{Mean}(D^2) = 2*(1 – r_p)$.

Although $\text{mean}(D^2)$ has a functional relationship with r_p, it is a misinterpreted measure as it is based on square differences. The square root of $\text{Mean}(D^2)$ ($\text{SQRT}[\text{Mean}(D^2)]$) apparently returns the measure to the original units of measurement (i.e., standard deviation), however, it is also a misinterpreted measure. A more intuitive measure is the mean *absolute* differences between paired standard scores (MADZ). Previous research has pointed out the misinterpretation and the statistical shortcomings of root-mean-*square* measure compared to mean-*absolute* measure in the contexts of covariance calculation (Falie and David, 2010), assessing model performance error (Willmott and Matsuura, 2005), and variability measure (Gorard, 2005). Falie and David (2010) presented the following relationship between the two measures of variability: mean absolute differences (MD) and standard deviation (SD) (assuming normally distributed random variables):

(3) $\text{MD} = \text{SD} * \text{SQRT}(2/\Pi) \approx 0.80 * \text{SD}$.

Gorard (2005) also found that SD values are typically larger than MD values by about 20% (see Figure 1, p. 422), and concluded: “SD is always greater than MD, but there is more than one possible SD for any MD value and vice versa” (p. 421).

Both MD and MADZ are mean absolute deviation measures, while SD and $\text{SQRT}[\text{mean}(D^2)]$ are root mean square deviation measures. Thus, the magnitude relationship between SD and MD can be applied to the magnitude relationship between MADZ and $\text{SQRT}[\text{mean}(D^2)]$. Assuming normally distributed variables, Formula 4 presents the estimated MADZ as a function of r_p using Formula 2 and the 20% magnitude difference between MADZ and $\text{SQRT}[\text{mean}(D^2)]$ adopted from Formula 3:

(4) $\text{MADZ} = \text{SQRT}[(2/\Pi)*2*(1 – r_p)] \approx 0.80 * \text{SQRT}[2*(1 – r_p)]$.

Using Formula 4, Figure 1 presents the expected relationship between MADZ and r_p for positive correlation coefficient values. Figure 1 shows that for positive values of r_p between 0 and 0.6, the relationship between r_p and MADZ approximates a linear one. MADZ decreases from 1.13 to 0.72 as r_p increases from 0 to 0.6. This approximately linear relationship has an intercept value of 1.13 and a slope of -0.68, and can be illustrated as:

(5) $\text{MADZ} = 1.13 – 0.68 * r_p$.

Research Aims

Given the above theoretically driven expected relationship between r_p and MADZ, this research has three main goals: (1) To confirm the theoretical relationship between r_p and MADZ described in Figure 1, using empirical data (Study 1); (2) To examine the subjective interpretations of the relationship between r_p and MADZ that people who are either less or more statistically proficient have (Study 2); and (3) To compare the empirical and theoretical relationships between r_p and MADZ to people’s subjective interpretations.

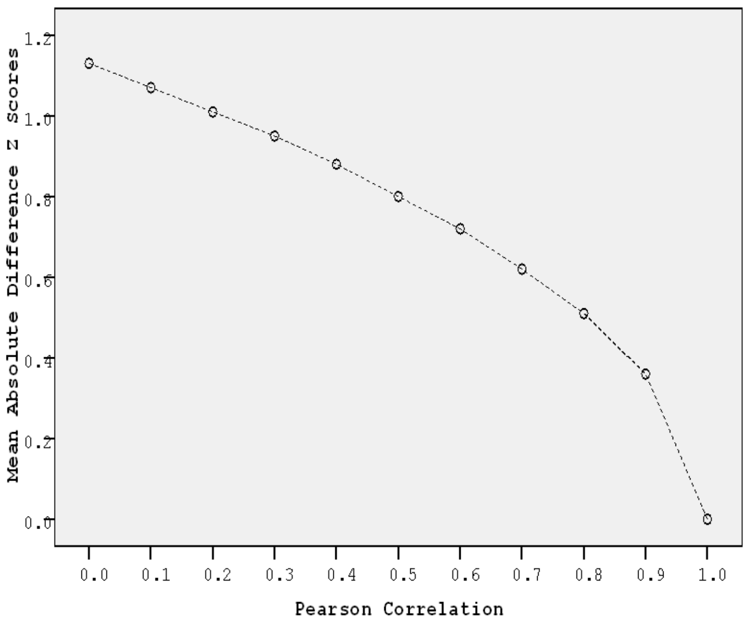

Figure 1
Theoretical relationship between the Pearson correlation coefficient (on the X axis) and the estimated mean absolute difference between paired standard scores (on the Y axis), assuming normally distributed variables

Study 1

Method

Three databases were used to estimate the empirical relationship between r_p and MADZ:
Database 1 included 17 data sets of empirical research findings in which 2–5 psychological variables were measured using self-administered questionnaires, yielding a total of 59 variables. The average descriptive statistics of the 17 samples were: 122 participants, average age of 25.3 (SD = 2.7), 59% were women. The Pearson coefficient of correlation was calculated between each pair of variables within each data set, yielding a total of 59 correlation coefficients that ranged between -.07 and .70, with an average of 0.23. For each correlation a respective mean absolute difference was calculated between the paired standard scores of the two variables. The respective MADZ values ranged between 0.62 and 1.18, with an average of 0.96.
Database 2 included 35 data sets: Seven cohorts of students in five academic departments at a higher education institution. The sample sizes of the data sets ranged between 44 and 157 students (M = 97). Students had two admission scores (a Psychometric Entrance Test score and a high school grade) and three criterion scores (1^{st}, 2^{nd}, and 3^{rd} year grade point averages – GPAs). A Pearson correlation was calculated between each of the two admission scores and the three GPAs, resulting in 174 correlation coefficients that ranged between -0.16 and 0.54, with an average of 0.24. The respective MADZ values ranged between 0.68 and 1.19, with an average of 0.95.
Database 3 included the grades of two cohorts of students in 15 and 12 required courses (the number of students having a grade in a course varied between 101 and 138). A Pearson correlation was calculated between the grades of each pair of courses, yielding 171 correlation coefficients that ranged between -0.02 and 0.64, with an average of 0.35. The respective MADZ values ranged between 0.64 and 1.15, with an average of 0.88.

Results and Discussion

For each of the three databases, MADZ was predicted from r_p using linear and polynomial equations. The measures of goodness-of-fit for the linear and polynomial equations were high and very similar for all three Databases (r^2=0.91, 0.62, and 0.92 for the linear equations of databases 1, 2, and 3, respectively; r^2=0.91, 0.62, and 0.93 for the polynomial equations of the respective databases). Thus, the relationship between r_p and MADZ was statistical and not functional for all three databases: for similar r_p values there were several corresponding MADZ values, representing the possible effect of additional factors, such as the distributions of the variables in each database. For parsimonious considerations, and following the similar goodness-of-fit measures of the linear and polynomial equations for each database, the following analysis will address the linear equations as representing the empirical relationship between r_p and MADZ, for r_p values between 0 and 0.6. The r^2 measures of the linear equations of the three respective databases were statistically significant, $F(1,57) = 557.5$, $p < .001$; $F(1,172) = 381.3$, $p < .001$; $F(1,169) = 1973.6$, $p < .001$.

Figure 2 presents the three regression lines predicting MADZ (on the Y axis) from the Pearson correlation coefficient (on the X axis) for the three databases. The respective intercept and slope values of the three databases were: 1.12 and -0.73 for Database 1; 1.08 and -0.56 for Database 2; 1.12 and -0.69 for Database 3. The figure clearly shows the proximity of the regression lines, especially for Databases 1 and 3.

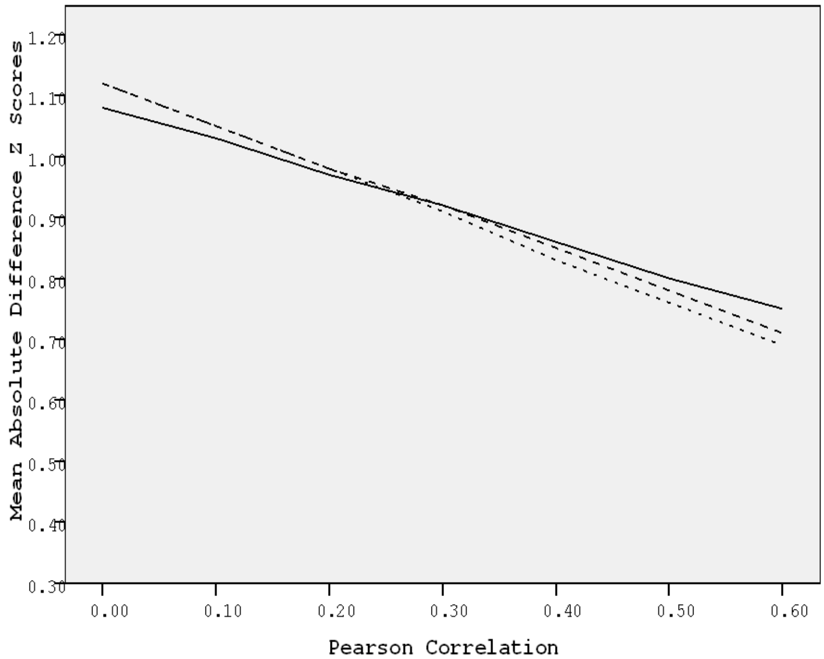

Figure 2
The regression lines representing the relationship between the Pearson correlation coefficient (on the X axis) and the respective mean absolute differences between paired standard scores (on the Y axis) for three databases: self-administered questionnaires (Database 1; the half-dashed line), students' admission and criterion scores (database 2; the full line), and students' grades in required courses (Database 3; the dashed line)

In order to examine whether the linear equations of the three databases are statistically different, the three databases were combined and a multiple linear regression predicted MADZ from: (1) the r_p values; (2) two dummy variables: one coded Database 1 as 1 and the other two as 0, and another dummy variable coded Database 2 as 1 and the other two as 0;

and (3) two product variables computed by multiplying r_p by each of the two dummy variables. The multiple linear regression yielded a very high multiple correlation (r^2 = .854, F(5, 403) = 464.35, p < .001); of the five predictors, r_p had the highest beta coefficient (-.986, t(403) = -29.59, p < .001), the dummy variable that coded Database 2 and its product were also statistically significant (beta = -.18, .18, t(403) = -3.66, 4.10, both p's < .001, respectively), and neither the dummy variable that coded Database 1 nor its product were statistically significant (beta = .01, -.04, t(403) = 0.17, -0.96, p = .87, .34, respectively).

Thus, the data can be characterized by two linear equations: one that characterizes Databases 1 and 3 (intercept=1.12; slope=-0.696; r^2= .92; F(1,228)=2665.1, p < .001), and another characterizing Database 2 (intercept=1.08; slope=-0.556; r^2= .69; F(1,172)=381.3, p < .001).

The theoretical prediction presented in Formula 5 closely matches the linear equation derived from Databases 1 and 3: The intercepts and slopes of both functions are similar (intercept values of 1.13 and 1.12; slope values of -0.68 and -0.70), and, as a result, the predicted MADZ values for r_p values in the range of 0 and 0.6 were also similar (mean absolute difference of 0.01). The intercept and slope coefficients of the linear function derived from Database 2 were slightly different (1.08 and -0.56), and, as a result, its respective predicted MADZ values differed slightly from the theoretical ones (mean absolute difference of 0.02).

The MADZ values predicted for r_p=0 both theoretically (Formula 5) and empirically (~1.1) may be higher than people's subjective interpretations. As the respective value of r_p=1 is MADZ=0, people might infer that the respective value of r_p=0 is MADZ=1. In addition, the theoretical and empirical slopes are between -0.56 and -0.7, whereas people might perceive the slope as -1, given the above two extreme coordinates of r_p and MADZ (0,1; and 1,0). The above paired values of r_p and MADZ (0,1; and 1,0) represent the following subjective function (henceforth "the subjective function"):

(6) $\text{MADZ} = 1 - r_p$.

If people estimate MADZ values using Formula 6, they would obviously underestimate the MADZ values for all non-negative r_p values. The aim of Study 2 was to examine the subjective interpretations of the relationship between r_p and MADZ among people who are more or less proficient in statistics. These interpretations will be compared with the theoretical and empirical relationships presented in Study 1.

Study 2

We hypothesized that people who are more or less proficient in statistics would interpret the relationship between r_p and MADZ similarly, according to the subjective function presented by Formula 6. We manipulated two independent variables that were hypothesized to either aid or hamper people's estimates of the slope and intercept of this subjective function. One independent variable presented half the participants with a correct estimate of the intercept (1.1), and the second variable manipulated the manner in which the various r_p values were presented: either ordered linearly or non-linearly. It was hypothesized that participants would make more estimations according to the subjective function presented in Formula 6 when the various r_p values are linearly ordered, and when the correct intercept value (MADZ=1.1 for r_p =0) was not presented to them. That is, presenting participants with r_p values that are non-linearly ordered or presenting them with the intercept value of 1.1 would hamper their attempts to provide MADZ estimates according to the subjective function presented in Formula 6. Following the typical non-negative values of r_p obtained in the empirical estimations of Study 1, and as study participants are confronted more with positive r_p values, Study 2 would examine the subjective interpretations of the relationship between r_p and MADZ for non-negative r_p values.

Method

Participants and Procedure. The sample comprised 174 participants: 99 1st year undergraduate students considered "less proficient with statistics" than the participants in the second sample, although all had successfully completed an introductory statistics course (87% female; average age of 23.5; *SD* = 1.9); and a second sample of 75 participants who were "more proficient with statistics". This sample comprised 22 Social Sciences graduate students and 53 members of a psychometric society. 72% of the second sample were female, and their average age was 36.6 (SD = 10.1). Of the 53 members of the psychometric society, 85% had at least a graduate degree. All participants answered an online questionnaire upon request.

Materials. The online questionnaire presented all participants with the following objective: Examination of subjective interpretations of the relationship between two variables. At the outset, participants were given a short reminder about the meaning of two statistical concepts—*standard score* and *the Pearson coefficient of correlation*. All participants were also presented with the zero mean absolute difference between each pair of standard scores when the correlation coefficient is 1. Participants were also reminded that MADZ increases as the correlation coefficient decreases (half the participants were presented with the MADZ value of 1.1 for correlation coefficient of 0). All participants were asked to estimate the size of the mean absolute differences between standard scores for 11 values of the Pearson correlation coefficient presented in a table. Participants were once again reminded that if the Half the participants were informed that the maximal MADZ value is 1.1 for r_p =0. Half the participants were presented with linearly ordered r_p values (1, 0.90, 0.80, 0.70, 0.60, 0.50, 0.40, 0.30, 0.20, 0.10, 0) whereas the other half were presented with non-linearly ordered r_p values (1, 0.95, 0.70, 0.65, 0.60, 0.45, 0.35, 0.25, 0.20, 0.15, 0). All participants were instructed to mark an MADZ estimate of r_p =1 as "0", and those participants receiving the MADZ value of 1.1 were instructed to mark it for r_p =0.

Design. The design included three independent variables: (1) *Proficiency*: Whether the participants were less acquainted with statistical concepts (undergraduate students) or more acquainted (graduate students or members of a psychometric society); (2) *Intercept*: Whether the intercept value (1.1) was or was not given; and (3) *Order*: Whether the r_p values were linearly ordered or not. Each participant was randomly assigned to one of the latter four conditions.

Results and discussion

Of the 174 participants, 20 supplied MADZ answers that were not an inversely monotone relationship of r_p. An additional four participants supplied MADZ values in a scale that deviated from the standard scores scale (e.g., MADZ values of 10-100 for r_p=0). An additional three participants who received the maximal MADZ value (1.1) for r_p=0, marked values of 2 or 3 for r_p=0. These 27 participants were removed from the analysis. Of the remaining 147 participants, 81 were undergraduate students (21, 18, 23, 19 participants in each experimental group) and 66 belonged to the more proficient group (12, 17, 17, and 20 participants in each experimental group).

Subjective Interpretations of the Relationship Between r_p and MADZ. A dichotomous variable indicated whether participants answered all 11 questions according to the subjective function (Formula 6) as hypothesized. Participants in the experimental conditions where the intercept value was not provided were coded as "1" only if all their 11 MADZ estimates corresponded to the values calculated by Formula 6; if at least one of the 11 MADZ values failed to match the calculated value, the participants were coded as "0". Participants that were provided with the 1.1 MADZ value for r_p=0 were coded as 1 if they provided: (a) the values calculated by Formula 6 for 10 r_p values, except r_p =0, which they were instructed to mark as 1.1; (b) the values calculated by Formula 6 for all r_p values, including r_p =0 where they disregarded the instruction to mark 1.1, and actually marked 1; or (c) the values calculated by Formula: MADZ=1.1*(1- r_p). Otherwise, these participants were coded as “0”.

As hypothesized, the percentages of participants providing the expected answers were higher when the r_p values were linearly ordered relative to non-linearly ordered. In three out of these four groups, 71%-86% of the participants provided 11 estimates that exactly matched the subjective function. An outlier was found for the more proficient group that received no intercept value; here only 47% of the participants provided the hypothesized values. Closer examination of this group revealed that out of the 17 participants, only eight provided the expected answers, while seven other participants provided MADZ values that were a linear function of r_p. These, however, ranged from 0 (for r_p =1) to 3 or 6 (for r_p =0). A possible reason for their providing these high values is the range of the standard scores (from -3 to 3) provided to the participants in the short explanation of standard scores. It is possible that these participants' answers related to the *maximal* absolute differences although they were asked about the *mean* absolute differences (failing also to consider that the maximal values of MADZ would be obtained for an r_p value of -1 and not 0). Disregarding these participants resulted in 80% of the remaining participants providing answers corresponding to the hypothesized subjective function. As hypothesized, providing participants with r_p values in a non-linearly ordered manner hampered their attempts to provide answers according to the subjective function: 17%-40% of the participants in these four conditions supplied the hypothesized answers.

Interestingly, 13 out of the 66 participants (20%) who received the MADZ value of 1.1 and were instructed to mark it as the corresponding value of r_p =0 provided a value of 1. Ten of these 13 participants were in the linearly-ordered condition, and all of them provided answers in accordance with the subjective function. These participants might have disregarded the instruction in order to provide the values predicted by the subjective function.

A three-way Analysis of Variance predicted the dichotomous variable of either providing a linear relationship or not from the three independent variables: proficiency, intercept and linear-order (although ANOVA usually requires continuous dependent variables, it was previously shown that in circumstances resembling the one in this study, such use is appropriate; Lunney, 1971, D'Agostino, 1971, Dey and Astin, 1993). Of the three main effects, only the order was statistically significant, $F(1,139) = 27.61$, $p < .001$, η^2=.166; neither the proficiency nor the intercept were statistically significant, $F(1,166) = 1.91, 0.00$, $p = .17, .95$, η^2=.014, .000, respectively. The three two-way interactions (intercept*order, intercept* proficiency, order*proficiency) were also not statistically significant, $F(1,139) = 0.36, 0.00, 1.58$, $p = .55, .96, .21$, η^2=.003, .000, .011, respectively. It was only the three-way interaction that was statistically significant, $F(1,139) = 6.11$, $p = .02$, η^2=.042, indicating a different pattern of two-way interactions of linearly-ordered and intercept among the two groups: Although in both groups the two linearly-ordered conditions resulted in higher proportions of participants providing linear relationships compared to the non-linearly ordered groups, for the more proficient participants, given the linearly-ordered pattern resulted in higher proportions of participants providing linear relationships when the intercept was given, while for those not given the intercept, the pattern was reversed. Among the undergraduate students the opposite was seen: having the linearly-ordered pattern resulted in lower proportions of participants providing linear relationships when the intercept was given, while for those not given the intercept, the pattern was reversed. However, taking into consideration the above explanation of the relatively high number of more proficient participants in the condition not receiving the intercept who gave unusually high MADZ values may raise doubts as to the meaningfulness of this result.

While the three-way interaction might be idiosyncratic for this sample, the main result of this study is that when presented with linearly-ordered values of r_p, most participants provided MADZ estimates that corresponded to the subjective function presented in Formula 6: MADZ $=1-r_p$. This main result applies to participants who are less or more proficient in statistics. In addition, providing participants with the intercept value did not seem to hamper their success in providing estimates in line with the subjective function. Most participants receiving the intercept values either supplied MADZ values that corresponded to the other r_p values or altered Formula 6 to modify the data [i.e., MADZ=1.1(1- r_p)], or simply ignored the instructions and used MADZ=1 for $r_p=0$.

Comparing the Results of Studies 1 and 2

Study 1 provided linear equations predicting MADZ values for r_p values between 0 and 0.6, which resembled the expected values predicted on the basis of Formula 5. Study 2 examined the subjective interpretations of this relationship and revealed that most participants, whether they were more or less proficient in statistics, gave MADZ estimates for various non-negative r_p values that reflected the subjective function (either the one presented in Formula 6 or a derivative of it), when r_p values were linearly-ordered. It was only when the participants were presented with r_p values in a non-linear manner that less than half of them provided estimates corresponding to this subjective function. Nevertheless, even under these conditions, in spite of the mathematical difficulties they encountered, about one-third of the participants successfully provided these expected values. Note also that the criterion for deciding that a participant was following the subjective function was rather strict: we demanded that all 11 estimates match the subjective function. Figure 3 presents the two estimated equations found in Study 1 and the subjective function that most participants in Study 2 followed when they were presented with linearly-ordered r_p values (MADZ=1- r_p).

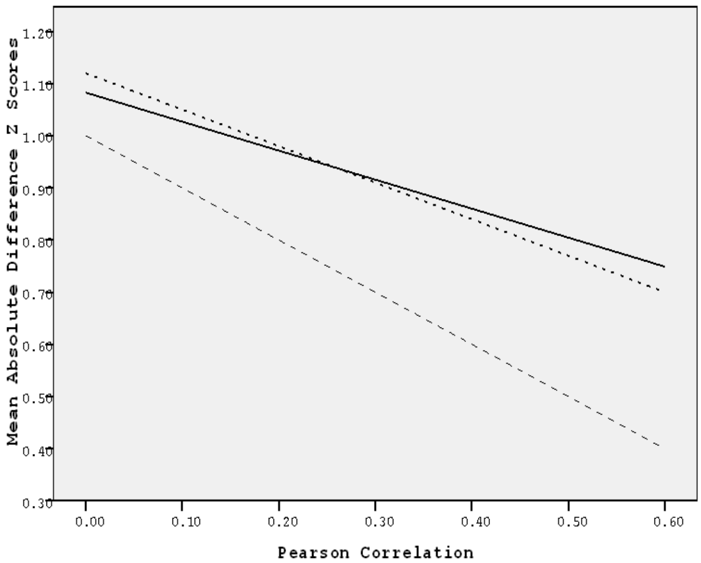

Figure 3
The bias of the subjective function (the dashed line) relating mean absolute differences between the paired standard scores as a function of Pearson correlation coefficient values between 0 and 0.6, relative to the empirical relationships (the half-dashed line representing self-administered questionnaires and students' grades in required courses; the full line representing students' admission and criterion scores)

Figure 3 clearly shows that the subjective function is biased relative to the empirical values. In the entire range of r_p values between 0 and 0.6, the subjective interpretation of the correlation represents an underestimation of the MADZ in a magnitude increasing from 0.1

for r_p=0 (subjective estimation of 1 relative to the correct value of 1.1), to 0.20 for r_p=0.3 (subjective estimation of 0.70 relative to the correct value of 0.90), 0.30 for r_p=0.5 (0.50 relative to 0.80), and 0.35 for r_p=0.6 (0.30 relative to 0.65).

General Discussion

This research developed a theoretical relationship between r_p values ranging from 0 to 0.6 and MADZ that could be approximately described as MADZ=1.13-0.68*r_p. This theoretical relationship closely matched respective relationships found in three empirical databases. In contrast, these similar theoretical and empirical relationships differed considerably from the respective subjective function that most people follow: MADZ=1-r_p. The subjective function is systematically biased with respect to the theoretical and empirical relationships: The MADZ values that people ascribe to the various non-negative r_p values are consistently lower than the values empirically found and those derived theoretically. For the range of r_p values that characterize empirical findings (0-0.6), this bias is considerable and it increases monotonically as the correlation increases. For example, a correlation coefficient of 0.5 would be interpreted subjectively as revealing half a standard deviation MADZ, although half a standard deviation actually corresponds to r_p=0.8. Thus, when interpreting r_p=0.5 people assign to it MADZ values that correspond to r_p=0.8. Stated differently, the actual MADZ value of r_p=0.5 (about 0.8) is ascribed by most people to r_p=0.2. Hence, if people were given the MADZ value that actually corresponds to r_p=0.5, they would estimate the Pearson correlation value as r_p=0.2. Thus, the subjective interpretations people hold about Pearson correlation values actually relate to much higher values. In effect, the correct interpretation of r_p values corresponds to subjective interpretations of much lower r_p values.

Several factors might explain the biased interpretation of the various values of the Pearson correlation coefficient. One possible factor is the manner in which the Pearson correlation is taught. Formulas that are typically used address the square of differences between raw scores or standard scores. These formulas do not provide teachers and students with a basis for developing a correct intuitive interpretation. Even the formula that relates r_p to the mean square difference between paired standard scores (Formula 1), which is rarely taught or used, involves an inverse linear transformation of the mean square differences between paired standard scores that results in a measure that has no straightforward intuitive appeal.

The biased interpretations might be perpetuated by the benefits it has for researchers trying to promote their empirical studies. The heuristically overestimation of positive correlation values contributes to the evaluation and appraisal of researchers' findings of various values of correlation coefficients. This "positive effect" might be self-preserving, as it "contributes" to the apparent scientific meaning of empirical findings, leaving very little room, if any, for actual interpretations presenting the emperor as (partly) outfitted. This "positive effect" might contribute to preserving the use of correlation coefficients and hamper the development of difference/discrepancy coefficients.

The results of Study 2 suggest that the heuristic interpretation of the correlation coefficients and its relationship to MADZ is a general phenomenon characterizing people with at least a basic understanding of the concepts of standard scores and the Pearson correlation coefficient. The similar findings for both groups of participants who are less or more proficient in statistics is consistent with heuristic thinking in other domains in which heuristic judgments of professionals were similar to those of the novice (e.g., Northcraft and Neale, 1987). The similar subjective interpretations of participants who are less or more proficient in statistics suggests that experience with statistics and with data does not immunize people against making subjective biased interpretations of the relationship between r_p and MADZ. As stated above, the current formulations of the Pearson correlation in both statistics textbooks and research literature do not assist experienced researchers to develop correct intuitions about the relationship between correlation values and mean absolute deviation between standard scores. Confronted with these formulations it is no wonder why experienced researchers, as well as undergraduate students, develop simple intuitive and heuristic impressions and judgments of correlation coefficients values that, as shown in this paper, are considerably biased.

De-biasing the interpretation of the Pearson correlation values is possible by teaching the empirical relationship between r_p and MADZ. Once the real interpretation of the r_p values is acknowledged in terms of MADZ, there might be more willingness to supplement (or even replace) the correlation coefficient by a measure of the difference between paired standard scores.

References

Cahan, S. (2000). Closeness to identity: A conceptual framework for the definition and interpretation of correlations. *Journal of Educational and Behavioral Statistics, 25*(4), 437-441.

D'agostino, R.B. (1971). A second look at analysis of variance on dichotomous data. *Journal of Educational Measurement, 8*(4), 327-333. doi:10.1111/j.1745-3984.1971.tb00947.x

Falie, D., & David, L. (2010). New algorithms of the absolute deviation covariance and correlation. *8th International Conference on Communications (COMM),* Bucharest. 57-60.

Falk, R., & Well, A.D. (1997). Many faces of the correlation coefficient. *Journal of Statistics Education, 5*(3), 1-18.

Gorard, S. (2005). Revisiting a 90-year-old debate: The advantages of the mean deviation. *British Journal of Educational Studies, 53*(4), 417-430.

Northcraft, G.B., & Neale, M.A. (1987). Experts, amateurs, and real estate: An anchoring-and-adjustment perspective on property pricing decisions. *Organizational Behavior and Human Decision Processes, 39*(1), 84-97.

Rodgers, L.J., & Nicewander, W.A. (1988). Thirteen ways to look at the correlation coefficient. *The American Statistician, 42*(1), 59-66.

Rovine, M.J., & von Eye, A. (1997). A 14th way to look at a correlation coefficient: Correlation as the proportion of matches. *The American Statistician, 51*(1), 42-46.

Willmott, C.J., & Matsuura, K. (2005). Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance. *Climate Research, 30*(1), 79-82.